PolyArc Fitter
― An approximation of a curve using line segments and arcs ―

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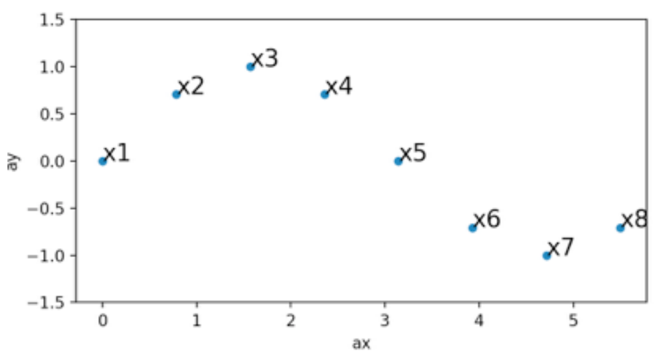
Abstract

Given a sequence of points on the plane, we propose an algorithm to approximate them by a curve consisting of line segments and circular arcs. It has a practical use in computer aided manufacturing [1-6], and our algorithm has already been used in ship building. In a typical pipeline, ship parts are designed by a CAD software and numerical control machines (NCM) are used to actually cut steel plates. Many NCMs are capable of cutting only line segments and circular arcs, so the designed curves have to be converted in such forms. Moreover, it is desirable to have as few segments as possible due to efficiency and physical limitations of the machine. Given a sequence of points on the plane, our algorithm produces a curve consisting of a small number of line segments and circular arcs which passes within a user specified neighbour from every point.

Introduction

Input

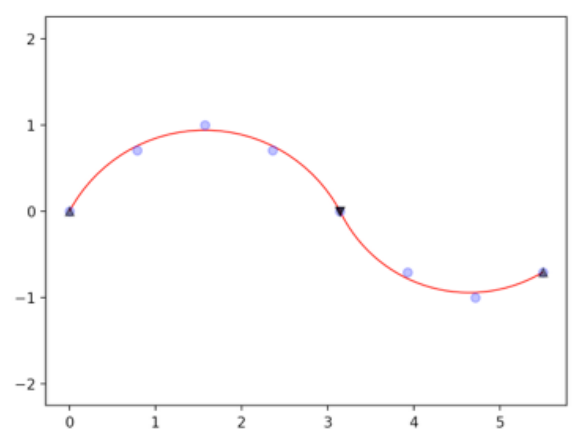
- a sequence of points $(x_1, y_1)\,(x_2, y_2),\dots,(x_n, y_n)$
- distance error tolerance
- angle tolerance



Output

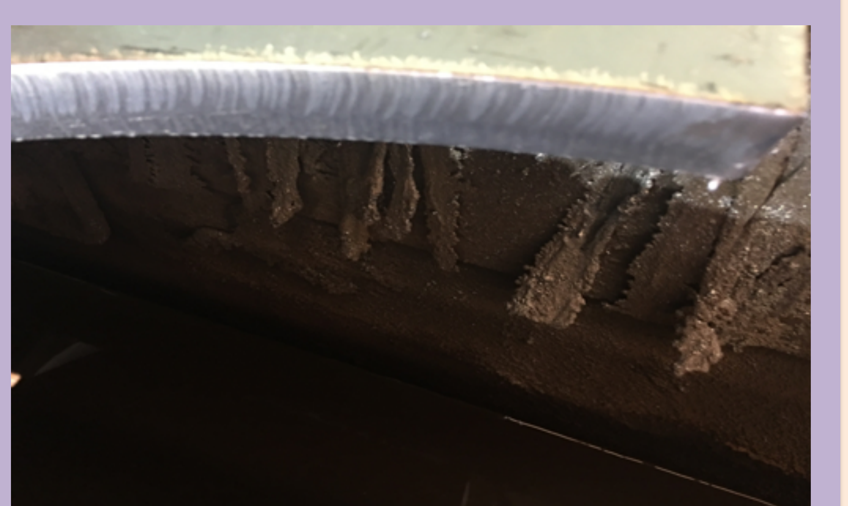
- sub sequence $(x_{i_1}, y_{i_1})\,(x_{i_2}, y_{i_2}),\dots,(x_{i_k}, y_{i_k})$
- arc or line segments connecting $(x_{i_j}, y_{i_j}), (x_{i_{j+1}}, y_{i_{j+1}})\ (j = 0, \dots, k-1)$
- arcs are specified by its

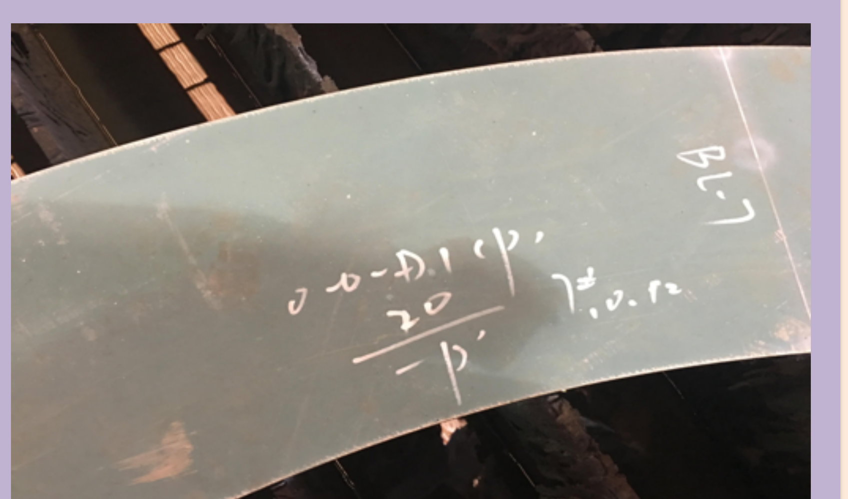
centre $c_{i_j} \in \mathbb{R}^2$
radius $r_{i_j} \in \mathbb{R}$
starting and ending angles $\theta_{i_j}, \phi_{i_j} \in [0, 2\pi)$



Industrial Use

The following pictures show actual metal plates cut by an NCM. Our algorithm is now used in a shipyard.



previous method used in a shipyard

our method

Algorithm

Step 1. Set j=1 and k=n.
Given end points (x_j, y_j) and (x_k, y_k), find an arc to fit the sequence $(x_j, y_j), (x_{j+1}, y_{j+1}), \dots, (x_k, y_k)$
This is achieved by solving a least squares problem described below.
Step 2. Check if the found arc conforms the specified error tolerance.
If not, by a binary search, find the maximum k so that Step 2 succeeds.
Step 3. Set j=k and repeat the process.

Problem:

$(x_i, y_i)(i = 1, \dots, n)$:points of sequences
a, b, c :variables

minimize

$$\frac{1}{2}\sum_{i=1}^{n}(x_i^2 + y_i^2 + ax_i + by_i + c)^2$$

subject

$$x_1^2 + y_1^2 + ax_1 + by_1 + c = 0$$
$$x_n^2 + y_n^2 + ax_n + by_n + c = 0$$

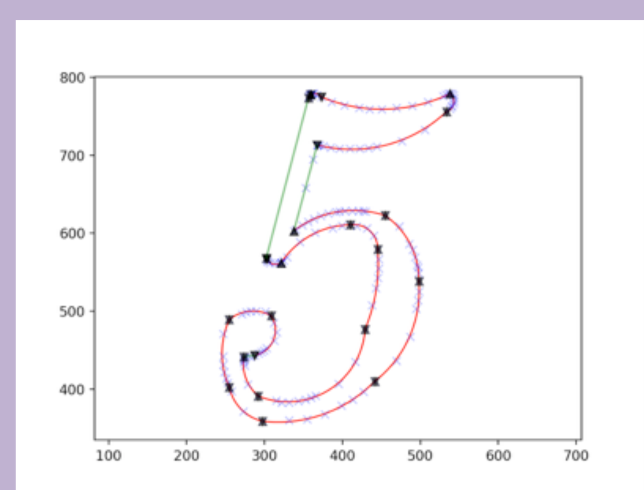
Solution of the problem:
By the Lagrange multiplier, we find

$$\begin{pmatrix} \boldsymbol{a} \\ \boldsymbol{\lambda} \end{pmatrix} = \begin{pmatrix} X^T X & Y^T \\ Y & \boldsymbol{O} \end{pmatrix}^{-1} \begin{pmatrix} -X^T \boldsymbol{p} \\ -\boldsymbol{q} \end{pmatrix}$$

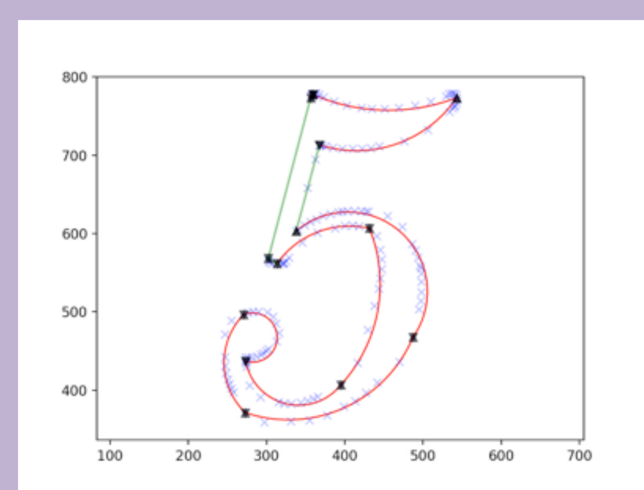
where

$$\boldsymbol{a} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}, \boldsymbol{\lambda} = \begin{pmatrix} \lambda_s \\ \lambda_e \end{pmatrix}, \boldsymbol{p} = \begin{pmatrix} x_1^2 + y_1^2 \\ x_2^2 + y_2^2 \\ \vdots \\ x_n^2 + y_n^2 \end{pmatrix}, \boldsymbol{q} = \begin{pmatrix} x_1^2 + y_1^2 \\ x_n^2 + y_n^2 \end{pmatrix}\ X = \begin{pmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ \vdots & \vdots & \vdots \\ x_n & y_n & 1 \end{pmatrix}, Y = \begin{pmatrix} x_1 & y_1 & 1 \\ x_n & y_n & 1 \end{pmatrix}$$

Result



maximum error = 2.0
18 arcs and 6 lines



maximum error = 8.0
9 arcs and 5 lines

Conclusion

- We can control the ratio between the number of units (line segments and arcs) and the error tolerance by specifying the parameters.
- Each unit takes the maximum length by binary search.
- Endpoints of each unit must coincide with some input points. Therefore, the output may not be optimal in terms of the number of units.
- Our energy function is not accurate for an arc in some cases (fig1).

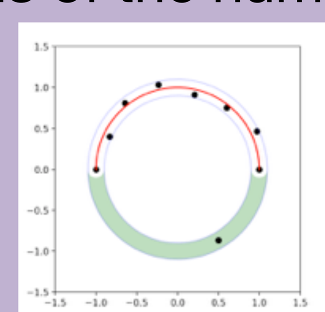


fig1

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