RIMS Research Project 2020

RIMS Review Seminar

Symmetry and Stability in Differential Geometry of Surfaces

Osaka City University and Online (Zoom)

February 14th – 16th, 2022

Titles & Abstracts

Jaigyoung Choe (KIAS, Korea)

Title: On the periodic Plateau problem

Abstract

The Plateau problem is to find the area minimizing surface spanning a given Jordan curve C. What if C is noncompact? We show that if C is a noncompact disconnected complete periodic curve, there exists a noncompact simply connected periodic minimal surface S spanning C. Uniqueness and embeddedness of S is proved under certain conditions on C.

Mikhail Karpukhin (Caltech, USA)

Title: Index of minimal surfaces in spheres and eigenvalues of the Laplacian

Abstract

Given a Riemannian surface, the study of sharp upper bounds for Laplacian eigenvalues under the volume constraint is a classical problem of spectral geometry going back to J. Hersch, P. Li and S.-T. Yau. The particular interest in this problem stems from the surprising fact that the optimal metrics for such bounds arise as metrics on minimal surfaces in spheres. In the first talk we will survey recent results on the subject with an emphasis on the role played by the index of minimal surfaces. The remaining two talks are devoted to the details behind the proof of the sharp bounds for all eigenvalues on the sphere and the projective plane, where the main ingredient is the classical algebro-geometric description of all minimal spheres in S^n via the so-called twistor correspondence.

Toshihiro Shoda (Kansai University, Japan)

Title: Moduli theory of minimal surfaces in flat tori

Abstract

The first study of the Moduli space of minimal surfaces in flat tori was done by Arezzo-Pirola who established the Moduli theory via the deformation theory of complex structures. They gave the existence theorem for minimal surfaces in flat tori. Other study was established by Ejiri via combining the theory of harmonic maps given by Schoen-Yau, Sacks-Uhlenbeck, the catastrophe theory, and the theory of complex Lagrangian submanifolds given by Cortés. In the process, the three geometric quantities, namely, the Morse index, the nullity, and the signature (a pair of non-negative integers) of a minimal surface play important roles, and algorithm to compute the three quantities are also established.

I would like to introduce the outline of them and their applications through the three lectures.

Takumi Gomyou (Nagoya University, Japan)

Title: Maximization of the first eigenvalue and embedding of a finite graph

Abstract

A graph can be realized in a Euclidean space by using eigenfunctions of the first nonzero eigenvalue of the graph Laplacian. Goering-Helmberg-Wappler considered the maximization of the first nonzero eigenvalue of the weighted graph Laplacian over all edge weights under a certain normalization. For an optimal edge weight of Goering-Helmberg-Wappler's problem, we obtain a graph embedding having a good geometrical property. We also consider variants of their problem.

Yoshiki Jikumaru (IMI Kyushu University, Japan)

Title: Geometry of hanging membranes from shell membrane theory and variational principle

Abstract

In this talk, we present differential geometric formulation of hanging membranes based on the equilibrium equations in the shell membrane theory and the variational principle. We also present the equations of hanging membranes in isothermic coordinates and some remarks on the application to architectural design. Topics in the talk are based on the joint work with Prof. Yohei Yokosuka (Kagoshima Univ.).

Eriko Shinkawa (AIMR Tohoku University, Japan)

Title: Symmetry and stability of anisotropic double crystals

Abstract

Double bubbles are a mathematical model of soap bubbles. The energy functional is the total area of the surface. On the other hand, when we think about a mathematical model of anisotropic substances like crystals, we need to consider the energy density function depending on the normal direction of the surface. The energy density function is called an anisotropic energy density function, and its sum (integral) over the surface is called anisotropic energy. In this study, we extend the double bubble problem to an anisotropic problem, that is, we minimize the anisotropic energy instead of the surface area. We derive the first and the second variation formulas of the energy functional. For $n = 1$ and a certain special energy density function, we classify the double crystals in terms of symmetry and the given areas. Also, we prove that some of the double crystals are unstable, that is, they are not local minimizers of the energy.