RIMS Research Project 2020

RIMS Symposium

The 4th International Workshop
Geometry of Submanifolds and Integrable Systems

Osaka City University and Online (Zoom)

February 20th – 23rd, 2022

Titles & Abstracts

Plenary Speakers

Fran Burstall (University of Bath)

Title: Eigenfunctions and representation theory

Abstract

A function on a Riemannian manifold is an eigenfunction if both it and its square are eigenvectors of the Laplacian. They can be used to construct harmonic morphisms and p-harmonic functions. In this talk, I shall explain how simple ideas from representation theory provide many examples of such eigenfunctions.

Emma Carberry (University of Sydney)

Title: Obtaining minimal surfaces in Euclidean 3-space by blowing up sequences of constant mean curvature tori

Abstract

I will consider sequences of constant mean curvature (cmc) immersions f_n of tori into $\mathbb{R}^3$ and explain under what circumstances a "blow up" (i.e. a rescaling of both the parameter for f_n and that of the ambient Euclidean space) produces a sub-sequence whose limit is a minimal surface immersion. These surfaces are studied via the sinh-Gordon and KdV integrable systems respectively, and the sequences of cmc tori have fixed spectral genus. In particular, certain optimally fast blow ups of cmc tori yield helicoids.

This is joint work with Sebastian Klein and Martin Schmidt (University of Mannheim).

Joseph Cho (TU Wien)

Title: Darboux deformations and transformations of discrete plane curves

Abstract

This talk explains how the integrability of semi-discrete isothermic surfaces leads to a definition of integrable smooth deformations and discrete transformations of discrete plane curves, and explores its connection to potential modified Korteweg-de Vries equations. This talk is based on the joint work with Wayne Rossman and Tomoya Seno (Kobe University).

Robert Kusner (UMass Amherst)

Title: Recent progress on the Canham and Willmore problems

Abstract

This is the first of two talks addressing the Willmore Problem (minimizing the bending energy $W := \int H^2$ among embedded surfaces in R^3 of genus g), and the related Canham Problem (where the isoperimetric ratio $v := 36\pi \mathrm{Vol}^2/\mathrm{Area}^3 \in (0,1)$ is prescribed as well):

I. More evidence for the conjecture that the Lawson surfaces with W less than 8π solve the Willmore problem: they are all W-stable (we show their Area Jacobi operator $\Delta + 2 + |A|^2$ has a spectral gap between eigenvalues –2 and 0), and they minimize W among surfaces with the same symmetries and topology (joint with Peng Wang).

II. Existence for the Canham problem involves constructing a comparison surface with W less than 8π and arbitrarily small isoperimetric ratio v; we do this by gluing $g+1$ small catenoidal bridges to the bigraph of a singular solution for the linearized Willmore equation $\Delta(\Delta+2)\phi = 0$ on the $(g+1)$-punctured sphere S^2; we also consider the action of the Möbius group on such solutions to study Canham minimizers in the small-v limit (joint with Peter McGrath).

Katrin Leschke (University of Leicester)

Title: Periodic discrete Darboux transforms

Abstract

We use an integrable system approach to discretise Darboux transformations of polarised curves via parallel sections of the associated family of flat connections. This allows to study closing conditions for both smooth and discrete polarised curves. In particular, we surprisingly obtain closed-form discrete parametrisations of Darboux transforms, closed Darboux transforms, the bicycle correspondences, and the closed bicycle correspondences of the discrete circle. This is joint work with Joseph Cho and Yuta Ogata.

Xiang Ma (Peking University)

Title: Mean curvature rigidity phenomenon and its extensions

Abstract

A theorem by Gromov asserts that for a hyperplane in the Euclidean space E^n, any smooth perturbation with compact support and nonnegative mean curvature H must be trivial (i.e. identical to the original one). We will start by presenting Souam's simple proof of this rigidity result using the tangency principle. Then we consider similar problems for the unit (hyper-)sphere with mean curvature $H = 1$ in E^n. Our main result says that when one perturbs the sphere only in a hemisphere, and the mean curvature H is no less than 1 for this smooth hypersurface after perturbation, then under quite natural conditions it must be congruent to the round sphere. On the other hand, if the fixed part of the sphere is only a small spherical cap, then there exist nontrivial perturbations on the complementary great spherical cap such that H is greater than 1 on the perturbed part. This is a joint work with Prof. Shibing CHEN (from USTC) and my student Shengyang WANG (from PKU).

Francisco Martín (Universidad de Granada)

Title: Translating annuli for the mean curvature flow

Abstract

Translating solitons, or translators for short, are self-similar solutions of the mean curvature flow which evolve by translation over a fixed direction. In this talk we will review some basic facts about these translators. Moreover, we will describe the construction of new examples of complete, properly embedded translating annuli in Euclidean space. The surfaces, which we call annuloids, are contained in vertical slabs and are not surfaces of revolution. We also provide new examples of complete, simply connected translators. This is a joint work with David Hoffman and Brian White.

Shigeki Matsutani (Kanazawa University)

Title: Submanifold Dirac operators based on the submanifold quantum mechanics, and their properties

Abstract

The submanifold quantum mechanics is a quantum system given by the restriction of the Schrödinger operator defined in $\mathbb{E}^3$ to a subspace associated with a lower dimensional submanifold. Due to the extrinsic curvature of the submanifold, there appears a potential in the operator,

which affects the Schrodinger particles. The geometrical potential was predicted by Jensen and Koppe (1971), and da Costa (1981). Recently its existence was experimentally confirmed by Onoe, Ito, Shima, Yoshioka, and Kimura (OISYK) 2012, for the electrons on a curved carbon system. In this talk, I will show the mathematical origin of the geometrical potential by using the Hörmander half-density form for the pre-Hilbert space, explain the experimental system of OISYK briefly and propose an algebraic construction of the submanifold differential operators on a submanifold embedded in $\mathbb{E}^n$ which reproduces the results of Jensen-Koppe-da Costa. By applying the construction to the Dirac operator, we obtain the submanifold Dirac operator. I will show some properties of the submanifold Dirac operator, e.g., the generalized Weierstrass representation, the index theorem of the normal bundles of the submanifold, and the geometrical realization of the quantum system appearing in the nonlinear integrable systems as the inverse scattering method.

Takashi Otofuji (Nihon University)

Title: Positive energy representations of affine algebras and Stokes matrices of the tt*-Toda equations

Abstract

We give a construction which produces a positive energy representation of the affine Lie algebra $\widehat{\mathfrak{sl}}_{n+1}\mathbb{C}$ from the Stokes data of a solution of the tt*-Toda equations. This is a joint work with Martin A. Guest.

Magdalena Toda (Texas Tech University)

Title: Fix boundary and free boundary problems for certain generalized Willmore energies

Abstract

The talk discusses elastic energies, and in particular the p-Willmore energy that we have studied over the past several years, along with its associated boundary value problems (fixed and free boundary problems). The techniques involved will combine results from variational calculus, classic geometric analysis, quaternionic theory, Lie groups and computational geometry.

Martin Traizet (Université de Tours)

Title: On the area of Lawson minimal surfaces in the 3-sphere

Abstract

Lawson has constructed, for each positive genus g, a closed minimal surface in the sphere S^3 whose area is less than 8π. I will explain how to construct these surfaces by integrable system methods (DPW) when the genus g is large. This approach yields fine estimates for their area. Suprisingly, the asymptotic expansion of the area when the genus goes to infinity involves the value of Riemann's zeta function at $z = 3$. I will explain the path from minimal surfaces in S^3 to values of the zeta function.

Joint work with Lynn Heller, Sebastian Heller and Steven Charlton.

Seiichi Udagawa (Nihon University)

Title: Spacelike CMC-surface in 3-dimensional anti-de Sitter space and Hill equation

Abstract

We first solve the sinh-Gordon equation. Next, we want to construct an oriented spacelike CMC-surface in anti-de Sitter space. We find that we must solve the Hill equation relating the problem. In case of minimal surface, the Hill equation turns out to be the Lame equation, which is solved explicitly.

Masashi Yasumoto (IMI Kyushu University)

Title: Discrete spacelike constant mean curvature surfaces in 3-dimensional Lorentzian space forms

Abstract

Using matrix-splitting theorems, Hoffmann derived a construction of discrete constant mean curvature (CMC for short) surfaces in the Eucludean 3-space. And the construction was extended to discrete CMC surfaces in 3-dimensional Riemannian space forms by Ogata and the speaker. This construction is now called the DPW method for discrete CMC surfaces.

In this talk we further extend the DPW method to discrete CMC surfaces in Lorentzian space forms, that can be regarded as a discrete version of construction of spacelike CMC surfaces developed by Brander-Rossman-Schmidt and Ogata. Our construction is similar to the previous result by Ogata and the speaker, but behaviors of the resulting surfaces are quite different. In particular, we analyze singular behaviors of discrete CMC surfaces in Lorentzian space forms obtained by our construction.

Short Communications

Rika Akiyama (Tokyo Metropolitan University)

Title: Variational problems for integral invariants of the second fundamental form of a map between Riemannian manifolds

Abstract

By using the idea of integral geometry, we define integral invariants of the second fundamental form of a map between Riemannian manifolds and construct a family of energy functionals including the bienergy functional. In this talk, we focus on some energy functionals defined by homogeneous polynomials of degree two among them and show their first variational formulae. From these results, we obtain alternative expression of the Euler–Lagrange equation of the bienergy functional and introduce the Chern–Federer energy functional whose Euler–Lagrange equation is a second-order partial differential equation.

This is a joint work with Takashi Sakai and Yuichiro Sato.

Sebastian Graiff-Zurita (Kyushu University)

Title: Fairing of planar curves by Log-aesthetic curves

Abstract

Log-aesthetic curves (LAC) constitute a family of planar spirals including the logarithmic spiral, Nielsen ' s spiral, Cornu spiral, and the circle involute, among others. This family is defined to best represent the properties observed by T. Harada et al. in a quantitative study of aesthetically pleasing curves used in industrial design. Many works have been written since then, mainly focused on the construction of the LAC for given constraints. In this work, as a first step to approximate a planar curve segment by Log-aesthetic curves, we show how to identify the parameters that uniquely define a LAC segment.

Yoshiki Kaneko (Waseda University)

Title: Solutions of the tt*-Toda equations from minuscule flag manifolds

Abstract

The tt*-Toda equations are introduced by Cecotti and Vafa. Guest, Its and Lin have studied solutions of the tt*-Toda equations, their asymptotic data and monodromy data form homomorphic data. When we consider the Dubrovin connections of minuscule flag manifolds, we can see that they are corresponding to the special solutions of the tt*-Toda equations.

Yoshiki Matsushita (Kyushu University)

Title: Geormtric shapes of maxfaces at singular points and Björling problem

Abstract

Maxface is a concept that is a slightly extended version of the maximal surfaces, and is a surface that may have singular points such as cuspidal edge and cuspidal cross cap.

In this talk, we will introduce the geometrical properties of maxfaces singularities and how to construct maxfaces with important singularities from the viewpoint of the Björling problem.

Ryosuke Odoi (Waseda University)

Title: Symplectic approach to the tt*-Toda equations and its application

Abstract

The tt* (topological-anti topological fusion) equations arose in the work of Cecotti and Vafa on supersymmetric quantum field theory, and the tt*-Toda equations are a special case of these equations. Solutions of the tt*-Toda equations can be considered as special kinds of harmonic maps from an open subset of $\mathbb{C}^*$ to the symmetric space $\mathrm{SL}(n+1,\mathbb{R})/\mathrm{SO}(n+1)$, or as certain isomonodromic deformations of meromorphic connections. Solutions can be parametrized by two kinds of data. One comes from asymptotic behaviour of the solutions, and the other one comes from monodromy including Stokes matrices. They correspond to each other via the Riemann-Hilbert correspondence. This correspondence can be considered as a transformation between two open dense subsets of the moduli space of the solutions. We will see that the transformation is symplectic and give an application of it.

Shota Shigetomi* (Kyushu University), Kenji Kajiwara (IMI Kyushu University)

Title: An explicit formula for isoperimetric deformation of discrete space curve with constant torsion angle

Abstract

Kaleidocycle is a closed linkage mechanism that is composed of identical tetrahedra jointed by hinges. It is known that the everting motion of Kaleidocycle can be regarded as the motion of a closed discrete space curve with a constant torsion angle. It is also known that the motion can be described by some integrable equations. In this talk, we give an explicit formula for isoperimetric deformation of discrete space curves with constant torsion angle. This formula can be regarded as an explicit formula of Kaleidocycle.