Speaker: Marcel Jackson

Title: Minimal signatures for undecidability of representability for relation algebras

Abstract: Tarski introduced his relation algebras in the 1940s as an algebraic model of the logic of binary relations. Subsequent work by many authors revealed the extraordinary complexity of these algebraic systems, with many negative results in the abstract theory, but also positive applications such: they provide the most common abstract framework qualitative reasoning models for example. Perhaps the most striking negative result is due to Hirsch and Hopkinson who showed in the late 1990s that the class of finite relation algebras isomorphic to concrete algebras of binary relations is nonrecursive: undecidability of the representability problem. Since that time there has been considerable interest in determining whether weaker systems might return to decidability of representability. Hirsch and Jackson (2012) showed that undecidability of representability still holds for lattice ordered monoids, Boolean monoids, and various other combinations including involuted monoids with meet (that is, Jónsson algebras, or allegories, in the sense of Freyd). The approaches introduced there led to various further weakenings that also exhibit undecidability. In this talk we give two results exploring the question of whether there is boundary of expressiveness where representability returns to being decidable. We show that the representability problem (as binary relations) is undecidable for complemented semigroups, which is a minimal subsignature of Tarski's for which undecidability holds; so, in one sense this is a boundary. However we also show that if one allows for term reducts (algebraic systems whose operations are term functions in Tarski's signature), then there is an infinite sequence of increasingly weaker signatures, all exhibiting undecidability of representability, but whose limit is the class of semigroups, where decidability is trivial, by way of the usual right regular representation (Cayley representation for semigroups). This is joint work with Robin Hirsch and Jaš Šemrl.