Deterministic Random Walk on Finite Graphs

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The rotor-router model, also known as the Propp machine, is a deterministic process analogous to a random walk on a graph [1]. Instead of distributing tokens to randomly chosen neighbors, the rotor-router model deterministically serves the neighbors in a fixed order by associating to each vertex a “rotor-router” pointing to one of its neighbors. The rotor-router model sometimes appears under the name of deterministic random walk, meaning a “derandomized, hence deterministic, version of a random walk.”

In this talk, we investigate the discrepancy at a single vertex between the number of tokens in the rotor-router model and the expected number of tokens in a random walk, for finite multigraphs. In case that the random walk is ergodic, reversible and lazy, we show that the discrepancy is $O(nm)$, where $n$ denotes the number of vertices, and $m$ denotes the total number of multiple edges [3]. For irreducible transition matrix $P$ in general, we show that the discrepancy is $O(\alpha^* n^2 m/(1 - \lambda^*))$, where $\lambda^*$ denotes the second largest eigenvalue of $P$, and $\alpha^*$ is a parameter defined by $P$ [2].

We also propose a new deterministic process, which we call functional-router model, in a similar fashion to the rotor-router model [4]. While the rotor-router is an analogy with random walks consisting of only rational transition probabilities using parallel edges, the functional-router can imitate random walks containing irrational transition probabilities. In fact, the functional-router can also emulate the rotor-router, thus the functional-router model is a generalization of the rotor-router model.

References


