2013 NIMS Hot Topics Workshops on

The 8th Jikji Workshop : Infinite Dimensional Analysis and Quantum Probability

Date **August 19(Mon) ~ 23(Fri), 2013**

Place **NIMS, Daejeon, Korea**

Invited Speakers :

- Luigi Accardi (Università di Roma Tor Vergata)
- Nobuhiro Asai (Aichi University of Education)
- Philiippe Biane (CNRS Universite Paris-Est)
- Pradip Kumar Das (Indian Statistical Institute)
- Fumio Hiai (Tohoku University)
- Sejong Kim (Louisiana State University)
- Yuh-Jia Lee (Natiional University of Kaohsiung)
- Franz Lehner (Graz University of Technology)
- Yongdo Lim (Sungkyunkwan University)
- Wojciech Mlotkowski (University of Wroclaw)
- Nobuaki Obata (Tohoku University)
- Habib Ouerdiane (University of Tunis El manar)
- Kimiaki Saito (Meijo University)
- Takashi Sano (Yamagata University)
- Tomoyuki Shirai (Kyushu University)
- Adam Skalski (Polish Academy of Sciences & University of Warsaw)
- Janusz Wysoczanski (University of Wroclaw)
- Anna Wysoczanska-Kula (University of Wroclaw)
- Hiroaki Yoshida (Ochanomizu University)

Local Organizing Committee :

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Program

	19 (Mon)	20 (Tue)	21 (Wed)	22 (Thur)	23 (Fri)
9:20~9:30	Opening				
9:30~10:20	Lehner1	Lehner3	Hiai1	Hiai3	9:30~10:10 Ouerdiane
					10:10~10:50 Shirai
10:30~11:20	Lehner2	Biane2	Hiai2	Skalski2	
					11:10~11:50 Das
11:40~12:30	Biane1	Biane3	Skalski1	Skalski3	11:50~12:20 Choi
12:30~13:30	Lunch	Lunch	Lunch	Lunch	Closing/ Lunch
13:30~14:20	Obata	Accardi	Excursion	Lee	
14:30~15:10	Janusz Wysoczanski	Saito		Anna Wysoczanska-Kula	
15:30~16:10	Kim	Yoshida		Mlotkowski	
16:10~16:50	Sano	Lim		Asai	
18:20~	Dinner	Banquet		Dinner	

Contents

Cumulants in noncommutative probability

Franz Lehner
Graz University of Technology

Abstract:

Cumulants (in the classical sense) were invented by Thiele at the end of the 19th century and have had many applications in statistics.
A century later other kinds of cumulants have been introduced to cope with different notions of noncommutative independence, like boolean and free cumulants and most recently monotone cumulants.
We give an introduction and general combinatorial theory of cumulants, covering the necessary lattice theory, set partitions, and some applications.

Concavification of free entropy

Philiippe Biane
CNRS Universite Paris-Est

Abstract:

I will review Voiculescu'e free entropy, and propose to modify slightly its definition in order to obtain a concave quantity. This modification allows in particular to solve the additivity problem for free entropy.

Spectra of Manhattan Street Digraphs for Synchronization of Coupled Oscillators

Nobuaki Obata
Tohoku University

Abstract:

A Manhattan street network is a digraph (directed graph), where the underlying graph is the $n \times m$ lattice and each edge is given direction in such a way that left and right directed horizontal lines are placed alternately, and so are up and down directed vertical lines.
In general, spectrum of a digraph contains useful information for the analysis of (deterministic or stochastic) dynamics on it.
We will report some results on spectral analysis for the preliminary case of $2 \times n$ lattice and apply to the study of synchronization of coupled oscillators.

References

[1] F. Comellas, C. Dalf'ó, M. A. Fiol and M. Mitjana: The spectra of Manhattan street networks, Linear Algebra Appl. 429 (2008), 1823-1839.
[2] N. Obata: Spectra of Manhattan products of directed paths $P_n \# P_2$, Interdisciplinary Information Sciences 18 (2012), 43-54. DOI: 10.4036/iis.2012.43
[3] N. Obata and J. Rodriguez: in preparation (2013)

FB-independence?

Janusz Wysoczanski
University of Wroclaw

Abstract:

We present a construction of Hilbert space operators, indexed by partially ordered set, which generalize free and boolean independences. More properties of this construction is to be shown, intended for new notion of independence.

Relative Operator Entropy and Fidelity

Sejong Kim
Louisiana State University

Abstract:

We consider operator versions of quantum Tsallis relative entropy that involve the weighted matrix geometric mean and establish relationships with the fidelity of a pure state and an arbitrary state. We define the symmetrized operator entropy based on the Tsallis relative operator entropy and study its properties.

Operator monotone functions and Kwong matrices

Takashi Sano
Yamagata University)

Abstract:

In this talk, we study operator monotonefunctions on (0, 1) and show some difierences from those on (0, ∞) with consideration on Kwong matrices

$$K_f(t_1, \ldots, t_n) := \left[\frac{f(t_i) + f(t_j)}{t_i + t_j}\right].$$

On the multi–dimensional Favard theorem

Luigi Accardi
Università di Roma Tor Vergata

Abstract:

The problem of developing a satisfactory theory of multi–dimensional orthogonal polynomials has been open for several decades.
The root of this problem is related to the fact that the multi–dimensional extensions of Favard's theorem existing, up to now, in the literature are not
satisfactory for several reasons.

In a recent **paper with A. Barhoumi and A. Dhahri** we have proved that the theory of orthogonal polynomials in d variables ($d \in \mathbb{N}$) can be canonically identfied to the theory of **symmetric** interacting Fock spaces over $\mathbb{C}^d$. The proof exploits the properties of the **quantum decomposition of vector valued random variables.**

This result opens the way to the investigation of **algebraic invariants of probability measures on** $\mathbb{R}^d$.

Infinite dimensional analysis based on higher order derivatives of white noise

Kimiaki Saitô*
Meijo University

Abstract:

In this talk we present an infinite dimensional analysis based on higher order derivatives of white noise and its recent developments. This analysis is associated with the exotic Laplacians and the Gross-Volterra Laplacians with orthonormal bases consisting of generalized functions.

The domain of the exotic Laplacian consists of higher powers of the derivative of white noise that are not normal polynomials introduced by P. Lévy. The exotic Laplacian acting on white noise distributions connects with the Lévy Laplacian acting on functionals of higher order derivative of white noise by the second quantization of the adjoint operator of the higher order differentiation.

This relationship comes from a generalization of the Accardi-Smolyanov theorem in the higher order Cesàro mean and implies a stochastic analysis of an infinite dimensional stochastic process generated by the exotic Laplacian.

We also discuss the relationship between the exotic Laplacians and the second quantization of the Fourier transform. It implies an infinite dimensional stochastic process consisting from the product of independent Fractional Brownian motions associated with the space of the exotic Laplacian acting on the Fourier transform of exotic test functionals.

*This work was supported by JSPS Grant-in-Aid Scientific Research 24540149.

The free logarithmic Sobolev and transportation cost inequalities by time integration

Hiroaki Yoshida
Ochanomizu University

Abstract:

We consider the time evolution of compactly supported probability measures on $\mathbb{R}$ by the free Fokker-Planck equation with the potential. Then we find that the formulas of the time derivation of the 2-Wasserstein distance with the Brenier map and of the dissipation of the free entropy imply the free functional inequalities, involving free LSI and free TCI, by time integration in case of a strictly convex potential.

Weighted least squares means on NPC space

Yongdo Lim
Sungkyunkwan University

Abstract:

Complete nonpositively curved metric spaces also called Hadamard spaces or simply NPC spaces have many fundamental properties which make them an attractive geometric object to study. These spaces somewhat resemble Hilbert spaces which provide an interesting new setting to study and carry out averaging of convex functions. Recently the so called least squares mean defined
as the unique solution of the minimization problem.

$$\min_{x\in M}\sum_{i=1}^{n} w_i\delta^2(x,a_i)$$

where $a_i \in M$, $w_i > 0$, $\sum_{i=1}^{n} w_i = 1$ and M is an NPC space, received broader attention. We introduce a new weighted random walk converging to the weighted least squares mean.

Contraction theory in quantum information

Fumio Hiai
Tohoku University

Abstract:

The first part of my lecture is a survey talk on Hilbert's projective metric and the irkoff-Hopf contraction theorem in Banach spaces. The subjects are very old but do not seem very familiar. After introducing basic concepts and properties, I explain the Birkoff-Hopf theorem and its application to generalization of Perron-Frobenius theorem. Thomson's norm and base norm in Banach spaces are also discussed.
In the second part, my lecture turns to a contraction theory in matrix algebras in connection with quantum information. Contraction coefficients are introduced for a stochastic map (i.e., a trace-preserving linear map $\Phi : \mathbb{M}_n \to \mathbb{M}_{\tilde{n}}$ between matrix algebras such that the adjoint Φ^* is a Schwarz map) in terms of several quantum
information quantities such as quasi-entropies, monotone metrics, χ^2-divergences, etc. I show new as well as known results concerning relations among those contraction coefficients. in the classical case where stochastic maps reduce to column-stochastic matrices, the theory was well developed by Cohen et al., *Linear Algebra Appl.* **179**(1993) and by Choi-Ruskai-Seneta, *Linear Algebra Appl.* **208/209** (1994), however quite a few problems are still open in the quantum case.
In the third part, I talk about the reversibility (or sufficiency) of stochastic maps. Here, a stochastic map $\Phi : \mathbb{M}_n \to \mathbb{M}_{\tilde{n}}$ is said to be reversible for a set $\mathcal{C}$of states (or density matrices) if there is a stochastic map $\Psi : \mathbb{M}_{\tilde{n}} \to \mathbb{M}_n$ such that $\Psi(\Phi(\rho)) = \rho$for all $\rho \in \mathcal{C}$. A number of characterizations for reversibility were obtained by Hiai-Mosonyi-Petz-Bénny, *Rev. Math. Phys.* **23** (2011) and by Jenčová, *Rev. Math. Phys.* **24** (2012). Quantitative version of the reversibility problem for Φ on $\mathcal{C}$is also discussed by introducing certain distinguishability coefficients of Φ, the notion opposite to contraction coefficients.

Haagerup property for locally compact classical and quantum groups

Adam Skalski
Polish Academy of Sciences & University of Warsaw

Abstract:

The Haagerup property is one of the most intensively studied recently properties of locally compact groups, exhibiting both geometric and analytic character. I will discuss its various equivalent characterisations for a given group G, in terms of the existence of some particular
a) representation of G;
b) positive definite functions on G;
c) conditionally negative definite function on G;
d) affine action of G on a Hilbert space,
adding a characterisation in terms of 'typical' representations of G. I will then pass to the context of locally compact quantum groups, outlining respective definitions and explaining difficulties and subtleties one encounters in the quantum context. Several examples of classical and quantum groups with the Haagerup property will be introduced.
In the last part I will prove that a free product of discrete quantum groups with the Haagerup property has the Haagerup property and explain some connections with 'free products' of quantum Levy processes.

(partly based on joint work with Matt Daws, Pierre Fima and Stuart White).

Analysis of Complex brownian motion

Yuh-Jia Lee*
Natiional University of Kaohsiung)

Abstract:

A theory of generalized functions based on the complex Brownian motion $\{Z\ (t) : t \in \mathbb{R}\}$, for which each $Z(t)$ is $N(0,\ |t|)$ is established on the probability space $(\mathcal{S}'_c,\ \mathcal{B}(\mathcal{S}'_c),\ (dz))$, where $\mathcal{S}'$is the dual of the Schwartz space $\mathcal{S}$, $\mathcal{S}'_c$, the complexification of $\mathcal{S}'$, identified as the product space $\mathcal{S}'\times\ \mathcal{S}'$, $\mathcal{B}(\mathcal{S}'_c)$ the Borel field of $\mathcal{S}'\times\ \mathcal{S}_0$ and $v(dz)$ denotes the product measure $\mu_1(dx)\ \mu_1(dy)$. Using the representation of the complex Brownian motion

$$Z_t(x,y) = \frac{1}{\sqrt{2}}\left(\langle x, h_t\rangle + i\langle y, h_t\rangle\right),$$

where

$$h_t = \begin{cases} 1_{(0,t]}, & t > 0, \\ -1_{[t,0]}, & t < 0. \end{cases}$$

and employing the technique of white noise calculus initiated by Hida we analyze functionals of complex Brownian motion. To define generalized complex Brownian functionals, we adopt the space of CKS entire functionals as test functions. As applications, the stochastic integral with respect to a complex Brownian motion are defined and studied. The Itˆo formula for complex Brownian functionals is obtained and it is shown that the evaluation of stochastic integral with respect to a complex Brownian motion follows the rule of Stratonovich integral.

Key words and phrases. complex Brownian motion, white noise analysis.
* Joint work with Kuang-Ghieh Yen and supported by the National Science Council of Taiwan.

Construction of compact quantum groups related to functions on permutation

Anna Wysoczanska-Kula
University of Wroclaw

Abstract:

In 1989 S. L. Woronowicz suggested a method which allowed to construct the twisted SU(n) compact quantum groups (CQG). The method relies in general on a choice of a non-degenerate function on n-tuples. However, in many cases the related CQG becomes trivial (i.e. the C*-algebra is commutative). During the talk, we shall remind the method and shortly review the known examples of non-trivial CQG's comming from this construction. Then we shall try to classify the CQG's related to functions on permutations for n=3.

Distributions with binomial moments

Wojciech Mlotkowski
University of Wroclaw

Abstract:

We prove that if $p \geq 1$ and $-1 \leq r \leq p-1$ then the binomial sequence $\binom{np+r}{n}$is positive definite and therefore is the moment sequence of a probability measure $\nu(p, r)$. The support of $\nu(p, r)$ is contained in the interval $[0, p^p(p-1)^{1-p}]$. If $p>1$ isa rational number and $-1 < r \leq p-1$ then $\nu(p, r)$ is absolutely continuous and itsdensity function $V_{p,r}$can be expressed in terms of the Meijer G-function. In particular cases $V_{p,r}$ is an elementary function. Finally we prove that the binomial sequence $\binom{np+r}{n}$ is positive definite if and only if either $p \geq 1$ $-1 \leq r \leq p-1$ or $p \leq 0$, $p-1 \leq r \leq 0$. The measures corresponding to the latter case are reections of the former ones. We also show connections with the distributions having Fuss-Catalan moments $\binom{np+r}{n}\frac{r}{np+r}$. This is joint work with Karol A. Penson.

The Brenke-Chihara polynomials and q-deformed Bargmann measures.

Nobuhiro Asai
Aichi University of Education

Abstract:

I shall discuss the Brenke-Chihara polynomials generated from the Boas-Buck type generating functions (IDAQP, 2013).
Moreover, some connections with q-deformed Bargmann measures will be presented.

Unitarizing measures for the representation of Lie group and associated invariant difierential operators

Habib Ouerdiane
University of Tunis El manar

Consider a Lie group with a unitary representation into a space of holomorphic functions defined on a domain $\mathcal{D}$ of C^n and in $L^2(\mu)$, the measure μ being the unitarizing measure of the representation.
On finite dimensional examples, we show that this unitarizing measure is also the invariant measure for some difierential operators on $\mathcal{D}$.
In this talk we calculate these operators in the following elementary cases:
A) The commutative groups $(\mathbb{R},+)$ and $(\mathbb{R}-0,\times)$.
B) The three dimensional Heisenberg group.
C) The ane group on the real line.

Zeros of a certain Gaussian power series

Tomoyuki Shirai
Kyushu University

Abstract:

In this talk, we discuss the zeros of a certain real Gaussian process in the unit disk, which is a random power series defined as the limit of the so-called Kac random polynomials.
We give a pfaffian expression of correlation functions of the zeros of this Gaussian power series.

An Experimental Scheme To Generate Non-Classical States In Interacting Fock Space

Pradip Kumar Das
Indian Statistical Institute

Abstract:

An experimentally realizable scheme is considered for manipulating quantum states using a general superposition of products of interacting annihilation and creation operators. Application of such an operation on states with classical features introduces strong nonclassicality. This provides the possibility of engineering quantum states with nonclassical features.

Convergence of Weighted Sums in Noncommutative Probability Space

Byoung Jin Choi
Chungbuk National University

Abstract

We study various convergences of weighted sums of random variables in a non-commutative probability space of which the weights are in a von Neumann algebra.
More precisely, we study the weak law of large numbers, convergence rates and exponential convergence rates which give upper bounds of large deviation principle for weighted sums in noncommutative probability space. As applications, we first study convergence rates of weighted sums of random variables in the noncommu-tative Lorentz space, and secondly we study convergence rates of weighted sums of probability measures with respect to the free additive convolution. This is a joint work with my supervisor Un Cig Ji.

Mathematics Subject Classifications (2000): 46L53.

Keywords: noncommutative probability space, weighted sum, weak law of large numbers, convergence rates, exponential convergence rates, large deviation, free convolution.