

Rem. No co-ori. fiber for $P_{3,2}(1)$ 28

Lemma $H^k(CO^*(\tau^0(3,2), P_{3,2}(2)))$

$$\cong \begin{cases} \mathbb{Z} \text{ (gen. by } [\tilde{O}_0 + \tilde{O}_e] \text{)} & \underline{k=0} \\ \mathbb{Z} \oplus \mathbb{Z} \text{ (gen. by } \alpha_1 = -[\tilde{I}_0^0 + \tilde{I}_e^1] = [\tilde{I}_e^0 + \tilde{I}_0^1], \\ \alpha_2 = [-\tilde{I}_0^0 + \tilde{I}_e^0], \alpha_3 = [\tilde{I}_0^1 - \tilde{I}_e^1] \\ \text{with } 2\alpha_1 = \alpha_2 + \alpha_3 \text{)} & \underline{k=1} \end{cases}$$

Lemma $f: M^2 \rightarrow \mathbb{R}$ stable Morse fct.
 \uparrow closed

$$S_1^* \alpha_1(f) = 0$$

$$S_1^* \alpha_2(f) = -S_1^* \alpha_3(f) = \underline{\max(f) - \min(f)}$$

$$\text{in } Ho(\mathbb{R}; \mathbb{Z}) \cong \mathbb{Z}$$

Corollary

$\max(f) - \min(f)$ is a

cobordism invariant of f