5.3 Universal Homology Complex

$$C_*(\tau, P) = (C_k(\tau, P), \partial_k)_k$$

$$C_k(\tau, P) = \Big(\bigoplus_{\mathcal{F}:\ \text{co-ori.}} \mathbb{Z}\langle \mathcal{F} \rangle \Big) \oplus \Big(\bigoplus_{\mathcal{F}\ \text{NOT co-ori.}} \mathbb{Z}_2\langle \mathcal{F} \rangle \Big)$$

$$\partial_k : C_k(\tau, P) \to C_{k-1}(\tau, P)$$

$\delta_k : C_k(\tau, P) \to C_{k+1}(\tau, P)$ is NOT well-defined in general !

$$C^*(\tau, P) = \mathrm{Hom}(C_*(\tau, P), \mathbb{Z}_2)$$

$$CO^*(\tau, P) = \mathrm{Hom}\,(C_*(\tau, P), \mathbb{Z})$$

$C_*(\tau, P)$: NOT free

⇓

use free approximations

⇓

hyper-cohomology $\mathbb{H}^*$