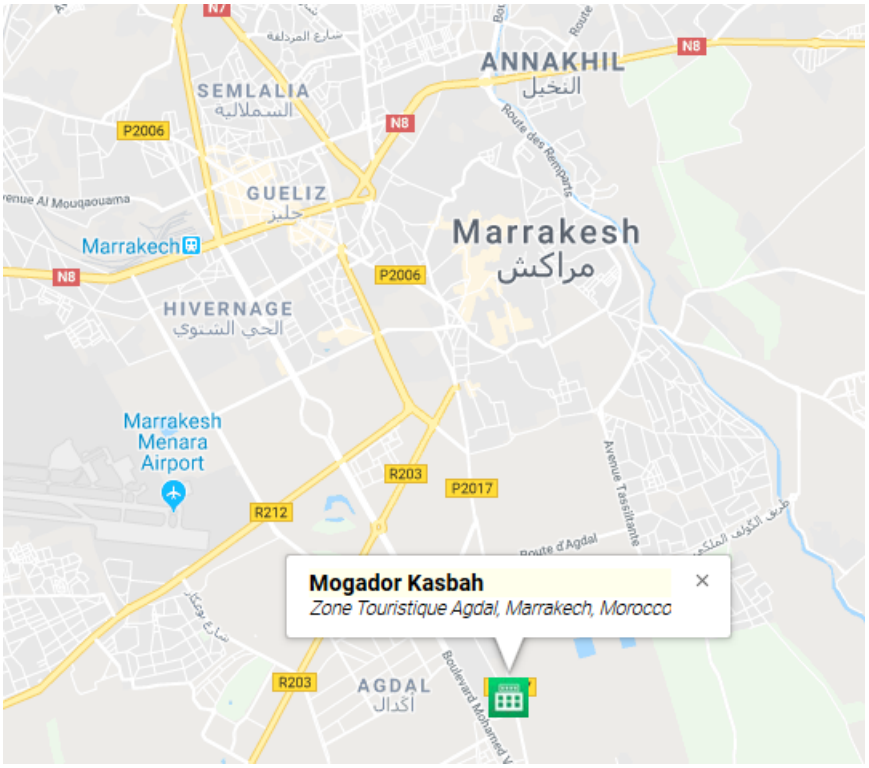
International Conference on Mathematical Methods in Physics

In memory of Professor Ahmed Intissar

Book of abstracts

1-5 April 2019 Marrakech
Morocco

The conference venue is
Ryad Mogador Kasba Hotel



Contents

Professor Ahmed Intissar (1951-2017)

At the Faculty of Sciences and Technics
Béni Mellal, 1995

After obtaining his PhD at the Massachusetts Institute of Technology (MIT) in 1985, Professor Ahmed Intissar returned to his native Morocco. Over the years, he supervised several PhD thesis, creating a school of mathematics which is worldwide recognized for fundamental contributions both in mathematics and mathematical physics. His students, friends and collaborators will always remember him for his teaching, his science and his charismatic personality.

Teams

Organizing committee

Abdelhamid Boussejra, Ibn Tofail University, Morocco
Paula Cerejeiras, Aveiro University, Portugal
Allal Ghanmi, Mohammed V University, Morocco
Yassine Hassouni, Mohammed V University, Morocco
Zouhaïr Mouayn (Chair), Sultan Moulay Slimane University, Morocco
Hassan Sami, Hassan II University, Morocco

Scientific committee

Abdelhamid Boussejra, Ibn Tofail University, Morocco
Hans G. Feichtinger, Vienna University, Austria
Jean P. Gazeau, Paris Diderot University, France
Zouhaïr Mouayn, Sultan Moulay Slimane University, Morocco
Ablamowicz Rafal, Tennessee Technology University, USA

Student Team

Khalid Ahbli, Ibn Zohr University, Morocco
Othmane El Moize, Ibn Tofail University, Morocco
Mohammed Mahboubi, Ibn Tofail University, Morocco
Ismail Ouaadi, Mohammed V University, Morocco
Ghayth Ouirdani, Sultan Moulay Slimane University, Morocco

Proceeding

The best papers presented at the conference and thematically connected with Clifford algebras or Clifford analysis and applications, Hopf algebras and quantum groups, geometric mechanics and symmetry, Clifford wavelets, etc will be submitted for publication in Advances of Applied Clifford Algebras (AACA) https://www.springer.com/birkhauser/physics/journal/6. The best papers related to Harmonic analysis, representation theory and quantization, Coherent states & wavelets, q-deformation, Orthogonal Polynomials and special functions, exact solvable systems, Spectral theory and quantum systems, will be submitted for publication in Complex Analysis and Operator Theory (CAOT) https://www.springer.com/birkhauser/mathematics/journal/11785.

List of registered participants

	Full Name	Affiliation and Email
A	S. Abumaryam	Sirte Univ., LIBYA, Souad22008@yahoo.com
	M. A. Abdelmoumene	Brothers Mentouri Constantine 1 Univ., ALGERIA, Adib.m.abdelmoumene@gmail.com
	Luís Daniel Abreu	Austrian Academy of Sciences, AUSTRIA, labreu@kfs.oeaw.ac.at
	Mariano A del Olmo	Valladoli Univ., SPAIN, olmo@fta.uva.es
	Khalid Ahbli	Ibn Zohr Univ., MOROCCO, ahbli.khalid@gmail.com
	Hélène Airault	Picardie Jules Verne Univ., FRANCE, Helene.airault@u-picardie.fr
	M. Ait Ben Haddou	Moulay Ismail Univ., MOROCCO, maitbenha@gmail.com
	Abdessamad Ait Brahim	Sultan Moulay Slimane Univ., MOROCCO, mouayn@gmail.com
	Pedro Alberto	Coimbra Univ., PORTUGAL, lena@mat.uc.pt
	Mohamed Amazioug	EPTHE, Ibn Zohr University, Agadir, MOROCCO, amazioug@gmail.com
	Tarik Amtout	Faculty of Sciences and Techniques Tangier, MOROCCO, tareqfstt@gmail.com
	Sama Arjika	Agadez Univ., NIGER rjksama2008@gmail.com
	Azzouz Awane	Hassan II Univ. , MOROCCO, a.awane@yahoo.fr
B	Fida Bahba	Monastir Univ., TUNISIA, fidabahba@gmail.com
	Fabio Bagarello	Palermo Univ., ITALY, fabio.bagarello@unipa.it
	Ali Baklouti	Sfax Univ., TUNISIA Ali.Baklouti@fss.usf.tn
	Peter Balazs	Austrian Academy of Sciences, AUSTRIA, peter.balazs@oeaw.ac.at
	Edwin Beggs	Swansea Univ., WALES, e.j.beggs@swansea.ac.uk
	Abdessamad Belfakir	Faculté des sciences -Rabat, MOROCCO, abdessamad.belfakir@um5s.net.ma
	Abdelhadi Benahmadi	Mohammed V Univ., MOROCCO, abdelhadi.benahmadi@gmail.com
	Abdessalem Benammar	Research Center in Industrial Technologies (CRTI), ALGERIA, Abs_benammar@yahoo.fr
	Jasel Berra-Montiel	Universidad Autonoma de San Luis Potosi, MEXICO, jasel.berra@uaslp.mx
	Christian Berg	Copenhagen Univ., DENMARK, berg@math.ku.dk
	Mohamed Berkani	Mohammed I Univ., MOROCCO, berkanimo@aim.com
	Stefan Berceanu	National Institute for Physics and Nuclear Engineering, ROMANIA, Berceanu@theory.nipne.ro
	Djahida Bouchefra	Melilab –Mila. ALGERIA, Djahida.bouchefra@gmail.com
	Abdelhamid Boussejra	Ibn Tofail Univ., MOROCCO, a.boussejra@gmail.com
	Danail Brezov	Architecture, Civil Engineering and Geodesy Univ., BULGARIA, danail.brezov@gmail.com
C	Paula Cerejeiras	Aveiro Univ., PORTUGAL, pceres@ua.pt
	A.Benaissa Cherif	Science & Technologu Univ., ALGERIA, Amine.banche@gmail.com
	Evaldo Curado	Centro Brasileiro de Pesquisas Físicas, BRAZIL, evaldo@cbpf.br
D	Briceyda B. Delgado	Southern Federal Univ., RUSSIA, briceydadelgado@gmail.com
	A. S. Diallo	Alioune Diop de Bambey Univ., SENEGAL, Abdoulsalam.diallo@uadb.edu.sn
	Kamal Diki	Politecnico di Milano, ITALY, kamal.diki@gmail.com
E	Amal El Hamyani	Mohammed V Univ., MOROCCO, amalelhamyani@gmail.com
	Othmane El Moize	Ibn Tofail Univ. MOROCCO, elmoize.othmane@gmail.com
	Omar El-Fallah	Mohammed V Univ., MOROCCO, elfallah@fsr.ac.ma
	Omar El Fourchi	CRMEF Rabat, MOROCCO, elfourchi@gmail.com
	Fouzia El Wassouli	Ibn Tofail Univ. , MOROCCO, elwassouli@gmail.com
	Abderrahman Essadiq	Ibn Tofail Univ., MOROCCO, aessadiq@yahoo.fr
	Pavel Exner	Charles Univ., CZECH REPUBLIC, exner@ ujf.cas.cz
F	Said Fahlaoui	Moulay Ismail Univ., MOROCCO, saidfahlaoui@gmail.com
	Jacques Faraut	Pierre & Marie Curie Univ., FRANCE, jacques.faraut@imj-prg.fr

	Full Name	Affiliation and Email
F	Nelson Faustino	Federal do ABC Univ., BRAZIL, nelson.faustino@ufabc.edu.br
	Hans G. Feichtinger	Vienna Univ., AUSTRIA, hans.feichtinger@univie.ac.at
	Milton d. S. Ferreira	Polytechnic Institute of Leiria, PORTUGAL, Milton.ferreira@ipleiria.pt
G	Francesco Gargano	Palermo Univ., ITALY, Francesco.gargano@unipa.it
	Jean P. Gazeau	Paris Diderot Univ., FRANCE, gazeau@apc.univ-paris7.fr
	Allal Ghanmi	Mohammed V Univ., MOROCCO, allalghanmi@gmail.com
H	Ali Hafidi	Moulay Ismail Univ., MOROCCO, Hafidiali28@gmail.com
	Iness Haouala	ESSTHS, TUNISIA, Iness.haouala@yahoo.fr
	Yassin Hassouni	Mohammed V Univ., MOROCCO, y-hassou@fsr.ac.ma
	Aziz Hilali	INPT, MOROCCO, hilali@inpt.ac.ma
I	Noureddine Imesmad	Ibn Tofail Univ., MOROCCO, Imesmad75@gmail.com
J	Vladimir Jacimovic	Montenegro Univ., MONTENEGRO, vladimir@jacimovic.me
K	Igor Kanatchikov	U. St Andrews, Scotland and Natl. Quantum Information Center, POLAND, ik25@st-andrews.ac.uk
	Tomas Kalvoda	Czech Technical Univ., CZECH REPUBLIC, Tomas.kalvoda@fit.cvut.cz
	Patrick Kayupe Kikodio	Ibn Tofail Univ., MOROCCO, kayupepatrick@gmail.com
	Uwe Kähler	Aveiro Univ., PORTUGAL, ukaehler@ua.pt
	Vladimir V. Kisil	Leeds Univ., UK, kisilv@maths.leeds.ac.uk
	Khalid Koufany	Lorraine-Nancy Univ., FRANCE, Khalid.Koufany@univ-lorraine.fr
	Abdelhak Kouraich	MOROCCO, akouraich@yahoo.fr
L	Fatima Zohra Ladrani	Higher Normal School of Oran, ALGERIA, f.z.ladrani@gmail.com
	Jerzy Lukierski	Wroclaw Univ., POLAND, Jerzy.lukierski@ift.uni.wroc.pl
M	M. Mahboubi	Ibn Tofail Univ., MOROCCO, Mohammedmahboubi04@gmail.com
	A. Makhlouf	Haute Alsace Univ., FRANCE, abdenacer.makhlouf@uha.fr
	Amel Mazouz	Djillali Bounaam Univ., ALGERIA, Mazouz.amel@yahoo.fr
	Mostafa Mbekhta	Lille Univ., FRANCE, mostafa.mbekhta@univ-lille.fr
	Alberto Molgado	Autonoma de San Luis Potosi Univ., MEXICO, Alberto.molgado@uaslp.mx
	Issam Mouayn	SCR & CARC, MOROCCO, imouayn@scrmaroc.com
	Zouhaïr Mouayn	Sultan Moulay Slimane Univ., MOROCCO, mouayn@gmail.com
	Z. Mouhcine	Mohammed V Univ., MOROCCO, zakariyaemouhcine@gmail.com
N	Samuel Nuugulu	Namibia Univ., NAMIBIA, snuugulu@unam.na
O	Anatol Odzijewicz	Institute of Mathematics, POLAND, aodzijew@uwb.edu.pl
	Youssef Omari	Faculté des sciences de Rabat, MOROCCO, omariysf@gmail.com
	Ismail Ouaadi	Mohammed V Univ., MOROCCO, ismail.ouaadi@gmail.com
	Achraf Ould Chaib	Ibn Tofail Univ. MOROCCO, achrafoualdchaib@gmail.com
	Ghayth Ouirdani	Sultan My. Slimane Univ., MOROCCO, ghaythouirdani@gmail.com
	Nadia Ourchane	Ibn Tofail Univ., MOROCCO, nadia.ourchane20@gmail.com
P	Lassi J. Päivärinta	Tallinn Univ. of technology, ESTONIA, Lassi.paivarinta@ttu.ee
R	Georgi Raikov	Pontificia Catolica Univ., CHILE, graikov@mat.uc.cl
S	Hassan Sami	Hassan II University, MOROCCO, samydavis_743@gmail.com
	Nobukazu Shimeno	Kwansei Gakuin Univ., JAPAN shimeno@kwansei.ac.jp
	Tomoyuki Shirai	Kyushu Univ., JAPAN, shirai@imi.kyushu-u.ac.jp
	Dmitry Shirokov	National Research Univ., RUSSIA, dm.shirokov@gmail.com
	Mohamed Sifi	Tunis El Manar Univ., TUNISIA, mohamed.sifi@fst.utm.tn
	Aneta Sliżewska	Institute of Mathematics Bialystok Univ., POLAND, anetasl@uwb.edu.pl
	Yahyeh Souleiman	Djibouti Univ., DJIBOUTI, yahyehlakios@gmail.com
T	Taoufiq Tahani	Lille 1 Univ., FRANCE, Taoufiq.Tahani@math.univ-lille1.fr
	S. Tremblay	Québec Univ ., CANADA, sebastien.tremblay@uqtr.ca
V	Apostol Vourdas	Bradford Univ., UK, a.vourdas@bradford.ac.uk
Y	H. A. Yamani	King Abdullah City for Atomic and Renewable Energy, SAUDI ARABIA, haydara@sbm.net.sa
Z	Slimane Zaiem	Batna 1 Univ., ALGERIA, zaim69slimane@yahoo.com
	Ahmed I. Zayed	DePaul Univ., USA, azayed@depaul.edu
	Ahmed Zeghal	Sultan My. Slimane Univ., MOROCCO, zeghal@yahoo.com
	Mohammed Ziyat	Mohammed V Univ., MOROCCO, Ziyatmohammed17@gmail.com

Abstracts

Harmonic analysis, representation theory and quantization

Plenary talks

Luís D. Abreu (Austria)

Planar electrostatics in higher Landau levels

Abstract. The beta-Weyl-Heisenberg ensemble will be suggested as a new Ansatz for electrostatics without confining potential, with the aim of overcoming the radial restrictions imposed on the confining potential by the structure of the Landau equation with a constant magnetic field. Particular cases include the 2D Coulomb gas model, the Ginibre ensembles, the polyanalytic ensembles, Laughlin's wave function, its lift to higher Landau levels and the Weyl-Heisenberg ensemble, recently introduced and studied in collaboration with Gröchenig, Pereira, Romero and Torquato. Further developments will be presented, based on joint work with Haimi and Luef and suggested by work of Duncan Haldane and the school of Ahmed Intissar.

Hélène Airault (France)

Matrix operators on symmetric Kaehler domains

Abstract. We show how to obtain the Laplacian and Ornstein-Uhlenbeck operators on some particular submanifolds of the Cartan domains II and III. This work extends the papers by Airault (Bull. Sci. math. 2012) and by Airault-Bouusejra (Bull. Sci. math. 2013)

Ali Baklouti (Tunisia)

Harmonic Analysis around monomial representations of exponential solvable Lie groups

Abstract. We discuss in this talk some problems related to a monomial representation $\tau = \mathrm{ind}_H^G \chi$ where G denotes an exponential solvable Lie group, H an analytic subgroup of G and χ a unitary character of H. The goal is to study The Plancherel formula and the algebra of G-invariant differential operators on the homogeneous space G/H.

Stefan Berceanu (Romania)

Differential geometry on homogeneous spaces attached to the Jacobi group

Abstract. The semidirect product G_n^J of the $(2n+1)$-dimensional Heisenberg group with the symplectic group of adequate dimension is known in Mathematics as the Jacobi group. The Jacobi group has many applications in Physics, beeng the group responsable for the squeezed states in quantum optics. Using the coherent state approach, we have determined the Kähler potential on the Siegel Jacobi ball and Siegel Jacobi upper half plane, which are homogeneous spaces attached to the Jacobi group, extending the results of Erich Kähler in the case $n = 1$. The invariant metric on the Kähler manifolds Siegel-Jacobi ball and Siegel-Jacobi upper half plane are balanced. Considering the real Jacobi group $G_1^J(\mathbb{R})$ as subgroup of submatrices of $\mathrm{Sp}(2, \mathbb{R})$, we study metric on the Siegel-Jacobi spaces invariant to the action of the Jacobi group. We calculate invariant metric on the 5-dimensional extended Siegel-Jacobi upper half plane.

Omar El-Fallah (Morocco)

Estimates of the singular values of Hankel operators on Bergman spaces

Abstract. Let $\mathbb{D}$ be a the unit disc of the complex plane and let A^2_ω be the Bergman space defined by

$$A^2_\omega := \{f \in Hol(\mathbb{D}) : \int_\Omega |f(z)|^2 \omega(z) dA(z)\},$$

where $\omega : \Omega \to (0, +\infty)$ is a continuous weight.

Let P the orthogonal projection from $L^2(\omega dA)$ onto A^2_ω. The Hankel operator associated with an anti-analytic symbol $\bar{b}$ is given by

$$H_{\bar{b}} f = \bar{b} f - P\bar{b} f, \quad f \in A^2_\omega.$$

In this talk we will discuss the boundedness, compactness of $H_{\bar{b}}$. We will also give some estimates of the singular values of $H_{\bar{b}}$. Other classes of operators will also be considered.

Khalid Koufany (France)

Conformally covariant bi-differential operators for differential forms

Abstract. The Rankin-Cohen bidifferential operators have attracted considerable attention in recent years particularly because of their applications to various areas including the theory of modular forms, covariant quantization, ring structures on representations spaces, branching law, ...

From the view point of representation theory the Rankin-Cohen operators are intertwiners in the branching law for the tensor product of two holomorphic discrete series representations of $SL(2, \mathbb{R})$: they give explicit intertwining operators from the tensor product of a given pair of irreducible representations into another irreducible representation (of higher weight).

In the first part of this talk I will explain the construction of a generalization of Rankin-Cohen bidifferential operators acting on $\mathcal{C}^\infty(V)$, which are covariant under the conformal group G of a simple real Jordan algebra V (see [1]).

In the second part I will construct these operators, replacing V by $\mathbb{R}^n$, the groupe G by $\mathrm{SO}_0(1, n+1)$ viewed as the conformal group of $\mathbb{R}^n$, and $\mathcal{C}^\infty(V)$ by differential forms (see [2]).

If time permits, I will also discuss the case of spinors ([3]).

References

1 S. Ben Said, J.-L. Clerc, K. Koufany. *Conformally covariant bi-differential operators on a simple real Jordan algebra.* International Mathematics Research Notices, May 2018, https://doi.org/10.1093/imrn/rny082.

2 S. Ben Said, J.-L. Clerc, K. Koufany. *Conformally covariant bi-differential operators for differential forms.* To appear in Communication in Mathematical Physics. arXiv:1809.06290.

3 S. Ben Said, J.-L. Clerc, K. Koufany. *Conformally covariant bi-differential operators for spinros.* In preparation.

Nobukazu Shimeno (Japan)

Matrix valued spherical functions on real semisimple Lie groups

Abstract. Let $G = KAN$ be a connected real semisimple Lie group of finite center. We define a notion of "minuscule K-types", which is a generalization of the notion of fine K-types for real split Lie groups due to Vogan and the notion of minuscule K-types for complex Lie groups. Under some additional conditions, there exists a first order invariant differential operators on the homogeneous vector bundle over G/K associated with a minuscule K-type. Moreover, the matrix valued elementary spherical functions on G for such a minuscule K-type can be expressed explicitly by Opdam's non-symmetric hypergeometric functions associated with the restricted root system of G/K. We also give some applications in harmonic analyisis for spherical functions.

This talk is based on joint work with Hiroshi Oda.

Ahmed I. Zayed (USA)

Sampling on a disc for a class of linear canonical transforms

Abstract. Sampling of bandlimited functions in different integral transform domains, such as Fourier, Hankel, and Legendre transforms, has been studied extensively either directly or indirectly through the initial-value problems which the kernels of the transforms satisfy. Some of these transforms have been extended to higher dimensions and more recently they have been extended to fractional orders. Sampling in higher dimensions is challenging because the distribution of the sampling points depend on the kernel of the transform as well as on the shape of the domain on which the transform is restricted. In this talk we will present a sampling theorem for functions that are bandlimited to a disc in the linear canonical transform domain, which includes the fractional Fourier transform as a special case.

Contributed talks

Jasel Berra-Montiel (Mexico)

Towards a representation of Loop Quantum Gravity in the Deformation Quantization approach

Abstract. We analyze the loop representation of quantum mechanics within the deformation quantization formalism. In particular, we construct the Wigner function and the star-product as a distributional limit of the Schrödinger representation for the Weyl algebra and we observe that this limit agrees with the Wigner function for Loop Quantum Cosmology. Finally, within our framework, we also derive a generalized uncertainty principle which is consistent to the ones usually obtained in theories assuming a fundamental minimal length in their formulation.

Fouzia El Wassouli (Morocco)

Wigner transform on line bundle on bonded symmetric spaces

Abstract. We generalized the Wigner transform on line bundle on bonded symmetric spaces. We also proved some it's properties and relations.

Milton dos Santos Ferreira (Portugal)

Gyroharmonic analysis and the continuous wavelet transform on the ball

Abstract. I will present the main results concerning gyroharmonic analysis on the ball with an arbitrary radius. Starting from the gyrogroup structure of the ball we construct the main tools of harmonic analysis on the ball: the unitary translation operator, the generalized eigenfunctions of the Laplace-Beltrami operator and the generalized Fourier-Helgason transform for the ball with hyperbolic geometry enclosed. With these tools in hand, we properly define a continuous wavelet transform on the ball by incorporating a class of radial relativistic operators together with motions on the ball given by Möbius transformations. We study the admissibility condition for this transform in order to have admissible wavelets and we give examples of admissible hyperbolic wavelets. The resulting wavelet transform is invertible whenever the mother wavelet satisfies a particular admissibility condition, which turns out to be a zero-mean condition. For large radiuses of the ball the continuous wavelet transform on the ball matches the usual continuous wavelet transform in $\mathbb{R}^n$.

Noureddine Imesmade (Morocco)

A characterization of the L^2-range of the Poisson transform on homogeneous line bundles over the matrix ball

Abstract. Let $X = G/K$ be a Riemannian symetric space of the noncompact type. In [1] Kaizuka solved the Strichartz conjecture on an image characterization for the Poisson transform P_λ on L^2-functions on the Furstenberg boundary, with regular parameter λ. In this talk we extend the above result to the case of a class of homogeneous line bundles over the hermitian symmetric space $SU(n,n)/S(U(n)\times U(n))$. This talk is based on a joint work with A. Boussejra.

References

1 A characterization of the L^2-range of the Poisson transform related to Strichartz conjecture on symmetric spaces of noncompact type, Adv. Math 303 (2016), 464-501.

Youssef Omari (Morocco)

Riesz bases of reproducing kernels in small Fock spaces

Abstract. We give a complete characterization of Riesz bases of normalized reproducing kernels in the small Fock space $\mathcal{F}^2_\varphi$, the space of entire functions f such that $fe^{-\varphi} \in L^2(\mathbb{C})$, where $\varphi(z) = (\log^+ |z|)^{\beta+1}$, $0 < \beta \leq 1$. The first results on this subject are due to Borichev-Lyubarskii who showed that φ with $\beta = 1$ is the biggest weight for which the corresponding Fock space admits Riesz bases of reproducing kernels. Later, such bases was characterized in Baranov et al. [1] for the case when $\beta = 1$. The present paper answers a question in [1] by extending their results for all parameters $\beta \in (0, 1)$. Our results are analogues to those obtained for the case $\beta = 1$ and those proved for Riesz bases of complex exponentials for the Paley-Wiener spaces. We also obtain a description of complete interpolating sequences in small Fock spaces with associated uniform norm. We characterize also complete interpolating sequences for the Fock spaces $\mathcal{F}^p_\varphi,\ 1 \leqslant p < \infty$, where $\varphi(z) = \alpha(\log^+ |z|)^2,\ \alpha > 0$. Our results are analogue to the classical Kadets-Ingham's $1/4$-Theorem on perturbation of Riesz bases of complex exponentials and they answers questions asked in [1].

References

1 A. Baranov, A. Dumont, A. Hartmann, K. Kellay. Sampling, interpolation and Riesz Bases in small Fock spaces. J. Math. Pures Appl. (9) 103 (2015), no. 6, 1358–1389.

2 A. Borichev and Yu. Lyubarskii, Riesz Bases of reproducing kernels in Fock type spaces, J. Inst. Math. Jussieu 9 (2010), no. 3, 449–461.

Achraf Ouald Chaib (Morocco)

The Poisson transform on homogeneous vector bundles over $Sp(n,1)/Sp(n) \times Sp(1)$

Abstract. In this talk I will give an image characterization of the Poisson transform on L^2-sections of a class of homogeneous vector bundles associated to finite irreducible representations of $Sp(n) \times Sp(1)$ which are trivial on $Sp(n)$. This extends previous results in [1] and [2] to the case of homogeneous vector bundle case over $Sp(n,1)/Sp(n) \times Sp(1)$. This talk is based on a joint work with A. Boussejra.

References

1 A. Boussejra, A. Intissar, L^2-concrete Spectral Analysis of the Invariant Laplacian in the Unit Complex Ball. J. Funct. Anal. 160 (1998), 115-140.

2 A characterization of the L^2-range of the Poisson transform related to Strichartz conjecture on symmetric spaces of noncompact type, Adv. Math 303 (2016), 464-501.

Nadia Ourchane (Morocco)

Characterization of the L^p-range of the Poisson transform on the Octonionic Hyperbolic plane

Abstract. In this talk, I will give a necessary and sufficient condition on eigenfunctions F of the Laplace-Beltrami operator on the octonionic hyperbolic plane $B(\mathbb{O}^2) = F_{4(-20)}/Spin(9)$ with eigenvalue $-(\lambda^2 + \rho^2)$ (λ a real number)to have an L^p-Poisson integral representations on the boundary $\partial B(\mathbb{O}^2)$, for $p \in]1, \infty[$. Namely, $F = P_\lambda f$ for some $f \in L^p \partial B(\mathbb{O}^2)$ if and only if it satisfies the following growth condition of Hardy-type:

$$\|F\|_{*,p} := \sup_{0 \leq r < 1} (1 - r^2)^{\frac{-\rho}{2}} \left(\int_{\partial B(\mathbb{O}^2)} |F(r\theta)|^p d\theta \right)^{\frac{1}{p}} < \infty.$$

Moreover, there exists a positive constant $\gamma(\lambda, p)$ such that for every $f \in L^p(\partial B(\mathbb{O}^2))$ the following estimates hold:

$$|c(\lambda)| \|f\|_p \leq \|P_\lambda f\|_{*,p} \leq \gamma(\lambda, p) \|f\|_p. \tag{1}$$

This extends the results in [1] and [2] for classical hyperbolic spaces to the case of the Octonionic Hyperbolic plane $F_{4(-20)}/Spin(9)$.
To prove the necessary condition, we use the techniques of singular integrals on the boundary $\partial B(\mathbb{O}^2)$ viewed as a space of homogeneous type in the sense of Coifman and Weiss [5]
To prove the sufficiency condition we follow the method we used in [3] and [4], to characterize Poisson integrals of L^p-functions on the Shilov boundary of bounded symmetric domains.
The talk is based on a joint work with A. Boussejra.

References

1 A. Boussejra, A. Intissar, Caractérisation des intégrales de Poisson-Szegö de $L^2(\partial \mathbb{B}^n)$ dans la boule de Bergman B^n $(n \geq 2)$, C. R. Acad. Sci. Paris I (1992), 309-313.

2 A. Boussejra, H. Sami, Characterization of the L^p-range of the Poisson transform in Hyparbolic Spaces $B(\mathbb{F}^n)$, J. Lie. Theory. 12 (2002), 1-14.

3 A. Boussejra, L^p-Poisson integrals representations of solutions of the Hua system on Hermitian symmetric spaces of tube type, J. Funct. Anal. 235 (2006), 413-429.

4 A. Boussejra, K. Koufany, Characterization of Poisson integrals for non-tube bounded symmetric domains, J. Math. Pures Appl. 87 (2007), 438-451.

5 R. Coifman, G. Weiss, Extension of Hardy spaces and their use inanalysis, Bull. Amer. Math. Soc. 83 (1977), 569-644.

Aneta Sliżewska (Poland)

Banach-Lie groupoids related to W^*-algebra

Abstract. In this presentation we will show that the groupoid of partially invertible elements and in particular the groupoid of partial isometries of a W^*-algebra have the structure of Banach-Lie groupoids. The relationship between these groupoids and Banach-Poisson geometry will be also disscussed.

References

1 A. Odzijewicz, A. Sliżewska, *Banach-Lie groupoids associated to W*-algebras*, Symplectic Geom. 14 (2016), no. 3, 687 - 736.

2 A. Odzijewicz, G. Jakimowicz, A. Sliżewska, *Banach-Lie algebroids associated to the groupoid of partially invertible elements of a W*-algebra*, J. Geom. Phys. 95 (2015), 108-126.

3 A. Odzijewicz, G. Jakimowicz, A. Sliżewska, *Fibre-wise linear Poisson structures related to W*-algebras*, J. Geom. Phys. 123 (2018), 385-423.

Clifford algebras, Clifford analysis and applications

Plenary talks

Paula Cerejeiras (Portugal)

$SU(3)$ symmetries on generalised ternary Clifford algebras

Abstract. In the last decade one observes an increasing interest in the study of more refined structures who model more accurately numerous physical problems. While the classic Dirac operator de- scribes structures with $SU(2)$-symmetries like the Fermions it is not very useful in other situations. Indeed, quite often problems in optics (polarisation filters) require analysis of $SU(3)$-symmetries. Other examples can be found in various scientific fields: the modified Ince equation in laser optics, the N-body problem, or theoretical descriptions of fundamental interactions based on the quark model. For all of these cases, the classic Dirac operator and related function theory do not provide a satisfactory representation. In this talk we are particularly interested in $SU(3)$-symmetries. This problem is nowadays mainly modelled by means of supersymmetry. However, we use another approach, based on ternary generalised Clifford algebras, or Z_3 -graded cubic algebras. We provide a complete algebraic de- scription of such algebras and develop the necessary analytic tools for a consistent function theory.

Uwe Kähler (Portugal)

Clifford-Krein modules with reproducing kernels

Abstract. Classic hypercomplex analysis is intimately linked with elliptic operators, such as the Laplacian or the Dirac operator, and positive quadratic forms. But there are many applications like the inversion of the crystallographic X-ray transform or or the study of null-solutions of the ultrahyperbolic Dirac operator which are closely connected with indefinite quadratic forms. Among other things this is due to the possibility of the underlying Clifford algebra to have a signature (p, q) and, therefore, to be linked to Pontryagin modules instead of Hilbert modules. Although in the majority of papers Hilbert modules are being used in this context they are not the right choice as function spaces since they do not reflect the induced geometry. In this talk we are going to show that Clifford-Krein modules are naturally appearing in this context. We take a particular look into the case of Clifford-Krein modules with reproducing kernels and use it to study interpolation and sampling problems.

Contributed talks

Dmitry Shirokov (Russia)

Method of averaging in Clifford algebras and applications

Abstract. We consider Reynolds operators of Salingaros group and some other operators in Clifford algebra and present a relation between these operators and projection operators onto fixed subspaces of Clifford algebras. Using these operators, we present general solutions of some algebraic systems of equations and systems of field theory equations. Using the method of averaging in Clifford algebra, we compute the elements that connect two sets of higher-dimensional gamma matrices and generalize Hestenes method of computing elements of spin groups to the case of arbitrary dimension.

Coherent states & wavelets

Plenary talks

Fabio Bagarello (Italy)

Extended kq-representation and bi-coherent states

Abstract. The standard kq-representation can be used to prove that a certain discrete subset of coherent states are complete in the Hilbert space of the square-integrable functions. This representation is based on the existence of common generalized eigenfunctions of the unitary operators

$$T_1 = e^{i\alpha\widehat{q}_0}, \qquad T_2 = e^{i\alpha\widehat{p}_0}, \tag{2}$$

where $\widehat{q}_0 = \widehat{q}_0^\dagger$ and $\widehat{p}_0 = \widehat{p}_0^\dagger$. Here, motivated by the appearance of pseudo-bosonic operators in several quantum systems, we introduce non self-adjoint position and momentum-like operators, and we use these to construct an extended kq-representation, sharing similar properties with the standard one. Then we use these properties to analyze the completeness of a set of bi-coherent states.

Peter Balazs (Austria)

Frames and Operators

Abstract. Frame theory is a very rich field in functional analysis, with significant impact in many other branches of mathematics, but also in theoretical physics and signal processing. For example the theory of continuous frame was developed in the context of coherent states. While frames have been used extensively for the description of functions, recently they have been used more and more for analyzing operators. We will focus on two aspects: multipliers and representation of operators.

Operators consisting of a transformation, a point-wise multiplication and a back-transformation play an important role in many scientific disciplines, like mathematics, signal processing and acoustics. In physics such so called quantization operators link classical and quantum mechanics. In this context these are so-called frame multipliers.

Those operators correspond to a diagonal operator in the coefficient domain. If we allow full matrices we can represent any bounded operator by such a system. Ina dual way, operators can be analyzed by the tensors of frames leading to a discretization scheme, which is for example used for the numerical solution of operator equations (in finite-element or boundary-element methods for example). In this talk we present the basic details of frame theory, talk about their generalizations and special cases (like wavelets). We will give an overview over the theory of multipliers, the representation of operators, their connections and properties of the tensors of frames.

Hans G. Feichtinger (Austria)

Coherent states, Gabor multipliers, and the Banach Gelfand Triple

Abstract. From the point of view of Abstract Harmonic Analysis (AHA) all we need for Time-Frequency Analyis (TFA) is a locally compact Abelian (LCA) group, which allows us to define the Short-Time or Sliding Window Fourier Transform (STFT) on the Hilbert space $L^2(G)$. For the case of the Euclidean group $G = R^d$ one can take the Gauss-function as a specific "window" or (thinking of the inversion formula, which allows to rebuild the given function f in $L^2(G)$ from the STFT) the "atom", i.e. write it as a (continuous) superposition of time-frequency shifted copies of the atom, the so-called coherent states. Gabor Analysis goes one step further, by replacing the continuous integral by an infinite sum. Anti-Wick operators (we also call them STFT-multipliers) are obtained by multiplying the STFT with some function "on phase space" before resynthesis, and in a similar way Gabor multipliers are obtained by multiplying the Gabor coefficients (indexed by some lattice in phase space) by some weight factors, before doing synthesis.

The goal of this talk will be to show how results about these operators can be compactly described using the Banach Gelfand triple (S_0, L^2, S_0'), also known as modulation spaces (M^1, L^2, M^∞). We view the Segal algebra S_0 as a Fourier invariant Banach algebra of test functions, and its dual as a space of "mild" distributions. They can be introduced making use only of Riemann integrals, without reference to Lebesgue measures and topological vector spaces.

The talk will focus on the usefulness of this perspective for questions in TFA, also indicating how this "rigged Hilbert space" situation allows to derive robustness properties of e.g. the Gabor frames and Gabor multipliers. Such results are not possible of one limits the view on the pure Hilbert space setting. Even more importantly one can ask whether the TFA and Gabor Analysis that can be realized over finite Abelian groups (and which can be implemented in a MATLAB environment) can be used to reliably approximate their continuous counterpart. Among others we will reveal an approach to the realization of discrete Hermite functions, based on experimental insight. As a matter of fact it coincides with a method proposed not too long ago by N. Cotfas. It is also the basis for what we consider to be an appropriate implementation of the fractional Fourier transform. The theoretical claims will be illustrated by numerical results.

Jean P. Gazeau (France)

Various generalizations of Glauber-Sudarshan coherent states for Quantum Optics

Abstract. Various generalisations of Glauber-Sudarshan coherent states for quantum optics will be presented in a unified way, with their statistical properties and their possible role in non-standard quantisations of the classical electromagnetic field. Some original statistical photon-counting aspects of Perelomov SU(2) and SU$(1,1)$ coherent states will be emphasized.

Vladimir V. Kisil (UK)

Hidden aspects of coherent states from group representation

Abstract. Coherent states is a widely-used tool in mathematical physics, which exists in several different flavours. The framework of group representation (aka Gilmore–Perelomov's coherent states) is a popular one with extensive coverage in many textbooks. Yet, some its aspects sadly escaped researchers' attention for a long time. For example, the following simple observation is very fruitful:

Lemma 0.0.1 (Analyticity of the coherent state transform). *[1] Let G be a group and dg be a measure on G. Let ρ be a unitary representation of G, which can be extended by integration to a vector space V of functions or distributions on G. Let a fiducial vector $\phi \in H$ satisfy the equation*

$$\int_G d(g)\, \rho(g)\phi \, dg = 0, \tag{3}$$

for a fixed distribution $d(g) \in V$. Then, any coherent state transform $\tilde{v}(g) = \langle v, \rho(g)\phi \rangle$ obeys the condition:

$$D\tilde{v} = 0, \qquad \text{where} \quad D = \int_G \bar{d}(g)\, R(g)\, dg\,, \tag{4}$$

with R being the right regular representation of G and $\bar{d}(g)$ is the complex conjugation of $d(g)$.

To illustrate the result we mention a simple corollary: the image spaces of the coherent state transform with n-th Hermite function consists of harmonic and true n-poly-analytic functions.

Benefits of systematic applications of Lemma 0.0.1 will be demonstrated in the talk by geometrisation of dynamics which can be obtained from squeezed states and Airy beams. These work was done in collaboration with Fadhel Almalkli [2].

References

1 Vladimir V. Kisil. Erlangen programme at large: an Overview. In S.V. Rogosin and A.A. Koroleva, editors, *Advances in Applied Analysis*, chapter 1, pages 1–94. Birkhäuser Verlag, Basel, 2012. `arXiv:1106.1686`.

2 Fadhel Almalki and Vladimir V. Kisil. Geometric dynamics of a harmonic oscillator, arbitrary minimal uncertainty states and the smallest step 3 nilpotent Lie group. *J. Phys. A: Math. Theor*, 52:025301, 2019. `arXiv:1805.01399`.

Contributed talks

Evaldo M. F. Curado (Brazil)

Optimizing Helstrom bound with non-standard coherent states

Abstract. In quantum information processing, when one tries to distinguish between two non-orthogonal states through some receiver device, there exists a quantum error probability. The latter is bounded below by a quantum limit named Helstrom bound. In this work we study and compare quantum limits for states which generalize the Glauber-Sudarshan coherent states, like non-linear and (modified) Susskind-Glogower coherent states. We show that for the latter ones the Helstrom bound can be significantly lowered.

Francesco Gargano (Italy)

Bi-coherent and Bi-squeezed states in non Hermitian Quantum Mechanics

Abstract. From a mathematical point of view, losing Hermiticity of the Hamiltonian of a physical system implies that the orthonormal basis of its eigenvectors must be replaced by two sets of biorthogonal states, no longer necessarily bases, but still complete (see [1]). In this context, in recent years, extended versions of the canonical commutation relations have been considered in details, and the related second-quantized framework produces the so-called pseudo-bosons. In this talk we first focus on the the notion of bi-coherent states (BCS) from a strictly mathematically oriented perspective. BCS can be considered as a non-hermitian generalization of coherent states, a class of quantum states playing a fundamental role both from a theoretical and an experimental point of view. Moreover, following the construction of the BCS, we generalize a somehow related class of states, introducing the bi-squeezed states (BSS). They can be considered as a suitable extension of squeezed states, introduced originally in quantum mechanics in order to describe non-linear processes such as optical parametric oscillations and four-wave mixing. This generalization of the squeezed states in the context of non-hermitian quantum mechanics arises from the introduction of suitable squeezing operators, [2]. Existence of BCS and BSS is in general not guaranteed, and the investigation on their existence is done for the well known Swanson model.

References

1 Deformed canonical (anti-)commutation relations and non Hermitian Hamiltonians Non-Selfadjoint Operators in Quantum Physics: Mathematical Aspects (New York: Wiley), Bagarello, Gazeau, Szafraniec Znojil,2015.

2 Bi-squeezed states arising from pseudo-bosons, Bagarello, Gargano, Spagnolo, , J. Phys. A: Math. Theor. 51 455204, 2018

Amel Mazouz (Algeria)

Husimi distribution: correspondence between quantum and classical dynamics

Abstract. The correspondence between classical and quantum mechanics is an important subject for the better inderstanding of quantum limits.

In particular, it is very important to investigate the correspondence between distribution functions in classical mechanics and in phase space representation of quantum mechanics as well as the Husimi function.

Topics dealt with include formulation of the Husimi representation and it application to analyses coherents and squeezed states of the 3D harmonic oscillator.

q-deformation, Hopf algebras and quantum groups

Plenary talks

Abdenacer Makhlouf (France)

q-Deformations and twisted algebraic structures

Abstract. A quantum deformation or q-deformation consists of replacing usual derivation by a σ-derivation or (σ, τ)-derivation in algebras of vector fields. The main example is given by Jackson derivative and lead for example to q-deformation of $\mathfrak{sl}_2$, Witt algebra, Virasoro algebra and also Heisenberg algebras (oscillator algebras). The description of the new structures gave rise to a structure generalizing Lie algebras, called Hom-Lie algebras or quasi-Lie algebras studied first by Larsson and Silvestrov. Since then various classical algebraic structures and properties were extended to the Hom-type setting. The main feature is that the classical identities are twisted by homomorphisms.

The purpose of my talk is to give an overview of recent developments and provide some key constructions and examples on Hom-algebras and BiHom-algebras. Moreover, I will describe (σ, τ)-differential graded algebra which generalizes the notion of differential graded algebra, and show example involving Clifford algebra.

References

1 V. Abramov, O. Liivapuu and A. Makhlouf, (q, σ, τ)-*Differential Graded Algebras.* Universe 2018, 4, 138; doi:10.3390/universe4120138 (2018)

2 F. Ammar and A. Makhlouf, *Hom-Lie Superalgebras and Hom-Lie admissible Superalgebras*, Journal of Algebra, Vol. 324, Issue 7, 1513Ð1528 (2010).

3 G. Graziani, A. Makhlouf, C. Menini, F. Panaite, *BiHom-associative algebras, BiHom-Lie algebras and BiHom-bialgebras*, Symmetry Integrability Geom. Methods Appl. **11** (2015), 086, 34 pages.

4 J. T. Hartwig, D. Larsson, S. D. Silvestrov, *Deformations of Lie algebras using σ-derivations*, J. Algebra **295** (2006), 314–361.

5 A. Makhlouf, S. Silvestrov, *Hom-algebras and Hom-coalgebras*, J. Algebra Appl. **9** (2010), 553–589.

Geometric mechanics, non-commutative geometry and symmetry

Plenary talks

Azzouz Awane (Morocco)

Polarized k-symplectic geometry

Abstract. We study various aspects and properties of polarized k-symplectic manifolds. We give a special interest to the study of Poisson structures subordinated to polarized $k-$symplectic structures of differentiable manifolds, and also, we introduce and study the notion of polarized vectorial Poisson manifold. Some properties and examples are given. The characteristic foliations of a polarized k-symplectic structure and the almost polarized $k-$symplectic structures are studied.

Danail Brezov (Bulgaria)

On the Complex Kinematics of Relativity

Abstract. The paper provides an alternative study on the proper Lorentz group $SO^{+}(3,1) \cong SO(3,\mathbb{C})$ in close analogy with the kinematical treatment of the group of spatial rotations. We first derive the former via projection from the Clifford group of $\mathbb{R}^3$, which may be considered a complex extension to the set of invertible quaternions $\mathbb{H}^{\times}$.

Thus, the Maurer-Cartan form yields a pure bi-quaternion valued analogue of the classical angular velocity de- scribing the action of $SO(3)$, which serves as a connection form restricted to time-parameterized one-subgroups.

The resulting complex kinematics of $SO^{+}(3,1)$ is far richer than the Euclidean one and yet, not too confusing at least in the decomposable setting, which is our main focus due to its great importance in theoretical physics. In particular, the associated geometric phase it yields consists of several distinct counterparts describing well known effects named after *Coriolis, Thomas, Hall* and *Sagnac*. We also consider a restriction to the three real forms, usually referred to in physical literature as "Wigner little groups" and discuss some dynamical aspects.

Finally, we comment on the way imposing different analyticity conditions on the homogeneous (Clifford) co- ordinates affects the corresponding projective picture and the underlying physics. One major advantage of our

description is its clarity and simplicity: each formula it provides comes with a clear geometric interpretation and we are able to easily pick nice examples from distinct fields such as mechanics, optics and particle physics.

Jerzy Lukierski (Poland)

New Approach to colour symmetries: Dirac equations for quarks and algebraic confinement

Abstract. We propose a modification of standard QCD description of the colour triplet of quarks describing quark fields endowed with colour degree of freedom by introducing a 12-component colour generalization of Dirac spinor, withbuilt-in Z_3 grading playing an important algebraic role in quark confinement. In colour Dirac equations the $SU(3)$ colour symmetry is entangled withthe Z_3-graded generalization of Lorentz symmetry, containing three 6-parameter sectors related by Z_3 maps. The generalized Lorentz covariance requires simultaneous presence of 12 colour Dirac multiplets, which lead to the description of all internal symmetries of quarks: besides $SU(3) \times SU(2) \times U(1)$,the flavour symmetries and three quark families.

The talk is based on the paper by:
R.Kerner and J.Lukierski, arXiv:1901.10936[hep-th].

Anatol Odzijewicz (Poland)

A family of integrable perturbed Kepler systems

Abstract. It is well known that the 3-dimensional Kepler system could be described, and thus integrated, as a system of four harmonic oscillators which are tied by a certain quadratic bound, e.g. see [1-4].

We also consider such systems of harmonic oscillators assuming however, that they are interacting in a non-linear way. This interaction is given by the Hamiltonians

$$\begin{aligned} H &= H_0(|\eta_1|^2, |\eta_2|^2, |\xi_1|^2, |\xi_2|^2) \\ &+ G_0(|\eta_1|^2, |\eta_2|^2, |\xi_1|^2, |\xi_2|^2)(\eta_1^k \eta_2^{-k} \xi_1^l \xi_2^{-l} + \eta_1^{-k} \eta_2^k \xi_1^{-l} \xi_2^l), \end{aligned} \tag{5}$$

where H_0, G_0 are arbitrary smooth functions of four real arguments and $k, l \in \mathbb{Z}$. Here for $z \in \mathbb{C}$ and $k \in \mathbb{Z}$ we assumed the convention

$$z^k := \begin{cases} z^k, & \text{if } k \geq 0 \\ \bar{z}^{-k}, & \text{if } k < 0 \end{cases} . \tag{6}$$

We will show that such Hamiltonian systems are integrable.

Next, after reduction to the submanifold of null-twistors and applying suitable symplectic diffeomorphisms, we show that these Hamiltonian systems are equivalent to a 3-dimensional perturbed Kepler systems written in the "fictitious time" representation, [4,5]. In such a way we obtain a family of 3-dimensional perturbed Kepler systems which are integrated by quadratures, see [5].

The solution of the Hamilton equations for a particular choice of (5) will be explicitly presented.

References

1 T. Iwai, *The geometry of the $SU(2)$ Kepler problem*, J. Geom. Phys. **7**, 507-535, 1990

2 M. Kummer, *On the Regularization of the Kepler Problem*, Commun. Math. Phys. **84**, 133-152, 1982

3 P. Kustaanheimo, E. Stiefel, *Perturbation theory of Kepler motion based on spinor regularization*, J. Reine Angew. Math. **218**, 204-219 (1965)

4 A.Odzijewicz, *Perturbed $(2n-1)$-dimensional Kepler problem and the nilpotent adjoint orbits of $U(n,n)$*, (to appear)

5 A. Odzijewicz, E. Wawreniuk, A. Sliżewska, *A family of integrable perturbed Kepler systems*, (to appear)

Lassi J.Päivärinta (Estonia)

Scattering of acoustic and electromagnetic waves by screens

Abstract. We study the scattering of electromagnetic (EM) and acoustic waves from thin objects that we call screens, more precisely , the scatterer is two dimensional object in three dimensional space. The incoming waves are assumed to be time-harmonic. A special difficult and interesting case is when the scatterer has corner points. These impose extra singularities for the fields.

The goal of this study is to bring new light to the theory of antennas. The basic experiments of antennas were done by Heinrich Herz in 1887. Herz constructed a dipole antenna which proved the existence of electromagnetic waves. We try, instead, to develop a theory for multipole antennas i.e. so called glover model. Some experiments made in Finland indicate that these might work better than dipole antenna.

This is a joint study with Sadia Sadique from Tallinn University of Technology.

Contributed talks

Edwin Beggs (Wales)

Geodesics in noncommutative geometry

Abstract. Geodesics play an important part in the theory of general relativity and differential geometry in general. There is an interpretation of classical geodesics in terms of bimodule connections (with different algebras on each side of the bimodule). This generalises to noncommutative algebras with calculi. The geodesics are described as paths in the state space of the algebra, and these states are given by Hilbert C*-modules in quite a simple fashion. The Hilbert C*-modules also provide a potential link to quantum theory. We shall look at examples from matrices and functions on finite groups, as well as first order deformations of classical manifolds. See https://arxiv.org/abs/1811.07601

Briceyda B. Delgado (Russia)

A right inverse operator for curl$+\lambda I$ and applications

Abstract. In this talk will be presented a general solution of the equation curl $\overrightarrow{\omega} + \lambda\overrightarrow{\omega} = \overrightarrow{g}$, $\lambda \in \mathbb{C}$, $\lambda \neq 0$ for an arbitrary bounded domain $\Omega \subset \mathbb{R}^3$ with a Liapunov boundary and $\overrightarrow{g}$ in certain class of integrable functions. The result is given in terms of classical integral operators of quaternionic analysis. Applications of this result are considered to Neumann boundary value problems for the equation curl $\overrightarrow{\omega} + \lambda\overrightarrow{\omega} = \overrightarrow{g}$ as well as to the nonhomogeneous time-harmonic Maxwell system for achiral and chiral media.

Kamal Diki (Italy)

The slice hyperholomorphic Cholewinski-Fock space on quaternions

Abstract. In this talk, inspired from the Cholewinski approach, we investigate a family of Fock spaces in the quaternionic slice hyperholomorphic setting as well some associated quaternionic linear operators. In a particular case, we reobtain the slice hyperholomorphic Fock space introduced and studied in 2014 by Alpay, Colombo, Sabadini and Salomon.

Alberto Molgado (Mexico)

Polysymplectic structures and general relativistic models

Abstract. A particular interesting issue within the canonical program for the quantisation of relativistic field theories is associated to the absence of a covariant Poisson structure at the classical level. In this direction, we will address a particular version for the analysis of classical field theories known as the polysymplectic formalism, which allows us to construct a well defined covariant Poisson bracket defined on the set of differential forms associated to the fields and its conjugate polymomenta. Though this formalism may be equivalent to the Lagrangian formalism at the classical level, the quantum program is far from being complete. In this talk, we will introduce the basics of the polysymplectic formalism for classical field theories and, at the classical level, we will explicitly present a couple of models associated to General Relativity. We will discuss certain advantages that the polysymplectic formalism may have in comparison to other approaches. Also, at the quantum level, we will introduce some ideas that we expect may shed some light on the ingredients that a covariant quantisation of relativistic field theories may need to incorporate.

Zaiem Slimane (Morocco)

Schrödinger Differential equation and Non-commutative geometry

Abstract. We obtain exact solutions of the 2D Schrödinger differential equation for central potentials in non-commutative complex space, using the Power-series expansion method. Hence, we can say that the Schrödinger equation in non-commutative complex space describes to the particles with spin (1/2) in an external uniform magnitic field. Where the noncommutativity play the role of magnetic field with created the total magnetic moment of particle with spin 1/2, who in turn shifted the spectrum of energy. Such effects are similar to the Zeeman splitting in a ordinare space.

Orthogonal Polynomials, special functions, exact solvable systems, non-linear systems

Plenary talks

Mariano A del Olmo (Spain)

Special functions, Lie algebras and rigged Hilbert spaces

Abstract. The aim of this communication is to present a unified picture of the connections between representations of Lie algebras, special functions, discrete and continuous bases and rigged Hilbert spaces.

Each class of orthogonal polynomials is a particular representation of a Lie algebra. This is the case of the Hermite polynomials and the Heisenberg-Weyl algebra or the associated Laguerre polynomials and the Spherical harmonics with $SO(3,2)$. Spaces supporting the representation of these associated algebras include discrete as well as continuous bases of which the matrix transformation is described by the orthogonal polynomials.

Hilbert spaces are not apt to describe these spaces as they do not have continuous bases. These spaces are quite often used by physicists, so that it would be necessary to introduce spaces allowing both discrete and continuous bases. They are the rigged Hilbert spaces.

Nobukazu Shimeno (Japan)

Matrix valued spherical functions on real semisimple Lie groups

Abstract. Let $G = KAN$ be a connected real semisimple Lie group of finite center. We define a notion of "minuscule K-types", which is a generalization of the notion of fine K-types for real split Lie groups due to Vogan and the notion of minuscule K-types for complex Lie groups. Under some additional conditions, there exists a first order invariant differential operators on the homogeneous vector bundle over G/K associated with a minuscule K-type. Moreover, the matrix valued elementary spherical functions on G for such a minuscule K-type can be expressed explicitly by Opdam's non-symmetric hypergeometric functions associated with the restricted root system of G/K. We also give some applications in harmonic analyisis for spherical functions.

This talk is based on joint work with Hiroshi Oda.

Hashim A. Yamani (Saudi Arabia)

Supersymmetry of tridiagonal Hamiltonians and their associated orthogonal polynomials

Abstract. We consider a given positive semi-definite Hamiltonian H that has a tridiagonal matrix representation in a basis $\{|\phi_n\rangle\}_{n=0}^{\infty}$: $H_{n,m} = \langle\phi_n|H|\phi_m\rangle = b_{n-1}\delta_{n,m+1} + a_n\delta_{n,m} + b_n\delta_{n,m-1}$. Associated with this Hamiltonian is a set of functions $\{f_n(E)\}_{n=0}^{\infty}$ that satisfies the three-term recursion relation: $Ef_n(E) = b_{n-1}f_{n-1}(E) + a_nf_n(E) + b_nf_n(E)$. If the Hamiltonian has a discrete part $\{E_\mu\}_\mu$ and a continuous part $\Omega_c(E)$, then the following orthogonality relation is satisfied $\delta_{n,m} = \sum_\mu f_n(E_\mu)f_m(E_\mu) + \int_{\Omega_c(E)} f_n(E)f_m(E)dE$. The set $\{p_n(E)\}_{n=0}^{\infty}$, defined by the relation $p_n(E)$ associated orthogonal polynomials. We then define a forward-shift operator A by its action on the basis: $A|\phi_m\rangle = c_m|\phi_m\rangle + d_m|\phi_{m-1}\rangle$. The parameters $\{c_n\}_{n=0}^{\infty}$ and $\{d_n\}_{n=0}^{\infty}$ can be derived from the sets $\{a_n\}_{n=0}^{\infty}$ and $\{b_n\}_{n=0}^{\infty}$, or directly from the polynomials as $c_n^2 = -b_n[p_{n+1}(0)/p_n(0)]$ and $d_{n+1}^2 = -b_n[p_n(0)/p_{n+1}(0)]$, where we set the ground state energy equal to zero. This in turn enables us to write the Hamiltonian in the form $H = A^\dagger A$. We then examine the properties of the supersymmetric partner Hamiltonian $H^{(+)} = AA^\dagger$. We show that it is tridiagonal in the same basis. The associated polynomials $\{p_n^{(+)}(E)\}_{n=0}^{\infty}$ turns out to be the kernel polynomials to $\{p_n(E)\}_{n=0}^{\infty}$. We relate the coefficients $\{a_n^{(+)}\}_{n=0}^{\infty}$ and $\{b_n^{(+)}\}_{n=0}^{\infty}$ to the coefficients $\{a_n\}_{n=0}^{\infty}$ and $\{b_n\}_{n=0}^{\infty}$.
We show that all these coefficients are derivable from the parameters $\{c_n\}_{n=0}^{\infty}$ and $\{d_n\}_{n=0}^{\infty}$, which turn out to be fundamental in this study. If, further, the Hamiltonian has a shape-invariance property, these parameters completely characterize the spectrum of the Hamiltonian and the hierarchy of its supersymmetric partners. They also enable us to derive the coherent states associated with the Hamiltonian. We illustrate these results with examples of known Hamiltonian having tridiagonal matrix representation in known bases.

Contributed talks

Pedro Alberto (Portugal)

Mathematical solutions of Coulomb type tensor interactions in relativistic quantum mechanics

Abstract. The Coulomb potential plays a crucial role in Physics, both in the classical, macro world and in the quantum, atomic and subatomic world. In this work we find the exact analytical solutions of the equation for a relativistic quantum particle of spin 1/2, i.e., the Dirac equation, for a generalized radial Coulomb potential with a tensor Lorentz structure. It turns out that the solutions can be expressed in terms of orthogonal polynomials, namely generalized Laguerre polynomials. This fact allows us to determine precisely the node structure of the respective wave functions in terms of the quantum numbers associated with the energy of the bound solutions. At the same time, because of the tensor character of the potential , it favors solutions with a particular alignment of the intrinsic spin and the orbital angular momentum of the quantum particle. We discuss also other special features of some solutions like infinite energy degeneracy.

Abdelhadi Benahmadi (Morocco)

Non-trivial 1d and 2d Segal-Bargmann transforms

Abstract. We consider the the univariate complex Hermite polynomials and we establish some interesting summation formulas including special generating functions and Mehler's formulas. Such results are then employed to introduce and study new integral transforms of Segal-Bargmann between some functional Hilbert spaces.

Zakariyae Mouhcine (Morocco)

An Integral Transform Connecting Spherical Analysis on Harmonic NA Groups to that of Odd Dimensional Real Hyperbolic Spaces

Abstract. The main aim of the present paper is to establish an integral transform connecting spherical analysis on harmonic NA groups to that of odd dimensional real hyperbolic spaces. Moreover, certain interesting integral identities for the Gauss hypergeometric functions have also been given.

Yahyeh Souleiman (Djibouti)

Numerical Analysis of a Sliding frictional contact problem with Unilateral Contact and Normal Compliance

Abstract. We consider a mathematical model which describes the equilibrium of a viscoelastic body in frictional contact with a moving foundation. The contact is modeled with a multivalued normal compliance condition with unilateral constraint and memory term, associated to a sliding version of Coulomb's law of dry friction. We present a description of the model, list the assumption on the data and derive a variational formulation of the problem, which is in a form of a system coupling a nonlinear equation for the stress field with a variational inequality for the displacement field. Then, we introduce a fully discrete scheme for the numerical approximation of the sliding contact problem. Under certain solution regularity assumptions, we derive an optimal order error estimate and we provide numerical validation of this result. Finally, we provide some numerical simulations in the study of a two-dimensional problem.

Mohammed Ziyat (Morocco)

Landau level coherent state transforms for quasi-tori

Abstract. The spectrum of the Laplcian operator on theta line bundle over the quasi-torus reduces to eigenvalues $\pi\ell$; $\ell = 0, 1, \cdots$, which are called Landau levels. This paper discusses the coherent state transform for each eigenspace associated with a Landau level. We construct an unitary transform valid for some factor of the eigenspace. A concrete form of the inverse formula for the proposed transform is also obtained.

Spectral theory and quantum systems, quantum information

Plenary talks

Pavel Exner (Czech Republic)

On Schrödinger operators exhibiting a parameter-dependent spectral transition

Abstract. In this talk we are going to analyze several classes of Schrödinger operators with potentials that are below unbounded but their negative part is localized in narrow channels. A prototype of such a behavior can be found in Smilansky-Solomyak model devised to illustrate that an an irreversible behavior is possible even if the heat bath to which the systems is coupled has a finite number of degrees of freedom. We review its properties and analyze several modifications of this model, with regular or strongly singular potentials, or a magnetic field, as well as another system in which $x^p y^p$ potential is amended by a negative radially symmetric term. All of them have the common property that they exhibit an abrupt parameter-dependent spectral transition: if the coupling constant exceeds a critical value the spectrum changes from a below bounded, partly or fully discrete, to the continuous one covering the whole real axis. We also discuss resonance effects in such models. The results come from a common work with Diana Barseghyan, Andrii Khrabustovskyi, Jiří Lipovský, Vladimir Lotoreichik, and Miloš Tater.

Jacques Faraut (France)

Horn's problem from a probabilistic point of view

Abstract. Let A and B be two $n \times n$ real symmetric or Hermitian matrices. Assume that the eigenvalues $\alpha_1, \ldots, \alpha_n$ of A are known, as well as the eigenvalues $\beta_1, \ldots, \beta_n$ of B. What can be said about the eigenvalues of the sum $C = A + B$? This is Horn's problem. The set of symmetric (resp. Hermitian) matrices X with spectrum $\{\alpha_1, \ldots, \alpha_n\}$ is an orbit $\mathcal{O}_\alpha$ for the natural action of the orthogonal group $O(n)$ on the space of $n \times n$ symmetric matrices (resp. of the unitary group $U(n)$ on the space of Hermitian matrices): $X \mapsto UXU^*$. Assume that the random matrix X is uniformly distributed on $\mathcal{O}_\alpha$, and, independently, the random matrix Y is uniformly distributed on $\mathcal{O}_\beta$. The problem is now to determine the joint distribution of the eigenvalues of the sum $Z = X + Y$. We know a formula for this distribution in case of Hermitian matrices. But in case of symmetric matrices we only know such a formula when the rank of B is equal to one.

Nelson Faustino (Brazil)

Fractional Fokker-Planck equations on lattices associated to Wilson-Dirac type operators

Abstract. The aim of this talk is centered around the fractional Fokker-Planck equation of the type

$$\begin{cases} \partial_t \Psi(x,t) = \frac{1}{2}(D_h^+ + D_h^-)\Psi(x,t) - \frac{t^{2H-1}}{2\Gamma(2H)}\Delta_h \Psi(x,t) & \text{if} \quad (x,t) \in h\mathbb{Z}^n \times (0,\infty) \\ \Psi(x,0) = \frac{1}{h^n}\delta_{0,x} & \text{if} \quad x \in h\mathbb{Z}^n \end{cases}$$

Hereby $D_h^{\pm}$ resp. Δ_h denote the forward/backward discretization of the Dirac operator $D = \sum_{j=1}^{n} e_j \partial_{x_j}$ resp. Laplace operator Δ on the lattice $h\mathbb{Z}^n$ (cf. [3]), whereas $0 < H < 1$ stands for the Hurst parameter (cf. [1]).

Its content is neatly summarized as follows:

- On the first part we will obtain possible conections between the aforementioned evolution problem and the probability distribution of Mittag-Leffler type $M_\nu(p,t) := p^{-\nu} M_\nu(pt^{-\nu})$ ($s,t > 0$ & $0 < \nu < 1$), described through the Laplace transform identities (cf. [6]):

$$\begin{aligned} \int_0^\infty e^{-sp} M_\nu(p,t) ds &= \sum_{k=0}^{\infty} \frac{(-pt^{-\nu})^k}{\Gamma(\nu k + 1)} \\ \int_0^\infty e^{-st} M_\nu(p,t) ds &= s^{\nu-1} e^{-ps^\nu}. \end{aligned}$$

- On the second part we will depict further subordination formulae encoded by the Mittag-Leffler probability distribution $M_\nu(p,t)$ that allows us to reformulate our evolution-type problem in terms of the integro-differential operator $\partial_t^\beta + (-\Delta_h)^\alpha$, carrying the Caputo derivative ∂_t^β ($0 < \beta \leq 1$) and the fractional powers of the discrete Laplacian $(-\Delta_h)^\alpha$ ($0 < \alpha \leq 1$).

Such approach may be seen as a continuation of the approach considered by the author in [4] and goes towards some recent results obtained by [7] and [2]. It relies on the replacement of the discrete heat type kernel encoded by the semigroup $\{\exp(t\Delta_h)\}_{t\geq 0}$ by the Mittag-Leffler distribution $M_\nu(p,t)$, which allows for applications in settings where the discrete Fourier transform on the toroidal manifold $\mathbb{R}^n/h\mathbb{Z}^n$ may be not the most appropriate tool to represent the symbol of the propagator.

The physical motivations besides are the so-called *lattice fermion doubling* gap carrying discrete counterparts of *continuum operators* (cf. [5]), and Wilson's regularization approach on the lattice (cf. [8]) endowed by the so-called Wilson-Dirac operator $\frac{1}{2}(D_h^+ + D_h^-) - \frac{t}{2}\Delta_h$. Indeed, for $H \to 1^-$ the time variable t may be interpreted as the Wilson parameter so that the solution of the aforementioned problem may be seen as a fractional regularization of the propagator $\left(\frac{1}{2}(D_h^+ + D_h^-) - \frac{t}{2}\Delta_h\right)^{-1}$.

References

1 Bender, C. (2003). *An Itô formula for generalized functionals of a fractional Brownian motion with arbitrary Hurst parameter.* Stochastic Processes and their Applications, 104(1), 81-106.

2 Ciaurri, Ó., Roncal, L., Stinga, P. R., Torrea, J. L., & Varona, J. L. (2018). *Nonlocal discrete diffusion equations and the fractional discrete Laplacian, regularity and applications.* Advances in Mathematics, 330, 688-738.

3 Faustino, N., Kähler, U., & Sommen, F. (2007). Discrete Dirac operators in Clifford analysis. Advances in Applied Clifford Algebras, 17(3), 451-467.

4 Faustino, N. (2018). *Relativistic Wave Equations on the lattice: an operational perspective.* arXiv preprint arXiv:1801.09340.

5 Friedan, D. (1982). *A proof of the Nielsen-Ninomiya theorem.* Communications in Mathematical Physics, 85(4), 481-490.

6 Gorenflo, R., & Mainardi, F. (2012). *Parametric subordination in fractional diffusion processes.* In Fractional Dynamics: Recent Advances (pp. 229-263).

7 Hahn, M., Kobayashi, K., & Umarov, S. (2011). *Fokker-Planck-Kolmogorov equations associated with time-changed fractional Brownian motion.* Proceedings of the American mathematical Society, 139(2), 691-705.

8 Wilson, K. G. (1974). *Confinement of quarks.* Physical Review D, 10(8), 2445.

Georgi Raikov (Chile)

Resonances and SSF singularities for a magnetic Schroedinger operator

Abstract. I will consider the 3D Schroedinger operator with constant magnetic field, perturbed by a rapidly decaying electric potential.

First, I will discuss the asymptotic behavior of the corresponding Krein spectral shift function (SSF) near the Landau levels which play the role of spectral thresholds. I will show that the SSF has singularities near these thresholds, which admit a fairly explicit description in terms of appropriate Berezin-Toeplitz operators. Further, I will introduce the resonances of the perturbed Schroedinger operator, and will discuss their asymptotic distribution near the spectral thresholds. I will show that under suitable assumptions on the perturbing potential, there are infinitely many resonances near every fixed Landau level, and the main asymptotic term of the corresponding resonance counting function is again related to the Berezin-Toeplitz operators arising in the description of the SSF singularities. Some extensions to Pauli operators with non-constant magnetic fields could be briefly outlined.

The talk is based on joint works with J.-F. Bony, V. Bruneau, C. Fernández, A. Pushnitski, and S. Warzel.

Tomoyuki Shirai (Japan)

Limit theorems for determinantal point processes

Abstract. The unitary group of size n together with the Haar probability measure is called Circular Unitary Ensemble of size n. All the eigenvalues lie on the unit circle in the complex plane and they can be regarded as a determinantal point process on the unit circle. It is also known that the scaled point processes converge to the determinantal point process associated with the so-called sine kernel as n tends to ∞. In this talk, we discuss a generalization of this fact and some related topics on determinantal point processes.

This talk is based on a joint work with Makoto Katori (Chuo University).

Sébastien Tremblay (Canada)

On generalized powers in one-dimensional supersymmetric quantum mechanics

Abstract. Representation for the general solution of the Sturm-Liouville equation as a spectral parameter power series (SPPS) was discovered ten years ago [1]. The power series is expressed in terms of a complete systems of recursive integrals generalized the system of powers $(x - x_0)^n$. These systems of recursive integrals, called generalized powers or ψ_0-powers, are expressed in terms of a nonvanishing function ψ_0 representing the ground state when we consider the Schrödinger's equation. In the last years, many works of Kravchenko et al. have shown that numerical methods based on SPPS representation is highly competitive in comparaison to the best available solvers. In this talk we will rather show new results on analytical properties of the generalized powers for the Schrödinger's equation and for supersymmetric quantum mechanics.

References

1 Kravchenko, V. V. and Porter, R. M. (2010), Spectral parameter power series for Sturm–Liouville problems. Math. Meth. Appl. Sci., 33: 459-468. doi:10.1002/mma.1205

Apostol Vourdas (UK)

Fermionic quantum computation and exterior calculus

Abstract. Grassmann's exterior calculus with its three operations meet, join and hodge star complement, is used for the representation of fermion-hole systems. The mathematical aspects of exterior calculus are interpreted physically in this context. Two different schemes that implement fermionic quantum computation, are proposed. The first scheme compares fermionic gates with Boolean gates, and leads to novel electronic devices that simulate fermionic gates. The second scheme uses a well known map between fermionic and multi-qubit systems, to simulate fermionic gates within multi-qubit systems.

Contributed talks

Abdessamad Belfakir (Morocco)

Generalized Heisenberg algebra: periodicity and finite representation

Abstract. We consider the Generalized Heisenberg Algebra(GHA) and the deformed GHA with finite representations. We provide the necessary restriction on the characteristic function of the periodic GHA and of the periodic deformed GHA and study particular examples. Then, we give a deformation of GHA raising operator of the Morse system and conclude that this system can be described by a periodic deformed GHA as it can be deduced from a nilpotente GHA.

Mohammed Berkani (Morocco)

On the B-discrete spectrum

Abstract. In this talk, we introduce the B-discrete spectrum of an unbounded closed operator. Then we prove that a closed operator has a purely B-discrete spectrum if and only if it has a meromorphic resolvent. An illustrating example of operator with purely B-discrete spectrum, is given by the Schrödinger operator with a constant magnetic field.

Abdoul Salam Diallo (Senegal)

Affine Szabó manifolds

Abstract. One says that a smooth manifold M of dimension n is a pseudo-Riemannian manifold of signature (p, q) if the tangent bundle T M is equipped with a smooth non-degenerate symmetric inner product g of signature (p, q) where $n = p + q$. Similarly one says that M is an affine manifold if T M is equipped with a torsion free connection ∇. One says that g is Szabó if the eigenvalues of the Szabó operators are constant on the pseudo-sphere bundles of unit timelike and spacelike vectors. We extend this concept from the pseudo-Riemannian to the affine setting to define the notion of an affine Szabó manifold. We establish some basic results concerning affine Szabó manifolds and exhibit examples of this structure.

Vladimir Jaćimović (Montenegro)

Quasiconformal geometry of a magnetic field

Abstract. We study the system of spin-$1/2$ particles in a magnetic field with state-dependent Hamiltonian. In this setup, the evolution of of magnetic field is described by a dy- namical system on the group $SU(1;1)$ of automorphisms of the unit disc. This approach employs known concepts and results from Complex Analysis and Hyperbolic Geometry to provide a mathematical explanation of some quantum effects.

References

1 Vladimir Jaćimović: Magnetization dynamics and geometry: coupled Möbius transformations, Rep. Math. Phys., vol. 81 (3), pp. 347 - 357 (2018).

Igor Kanatchikov (UK)

On the spectrum of a hypercomplex quantum operator related to the Yang-Mills mass gap

Abstract. We outline the approach of precanonical quantization fields which leads to a hypercomplex generalization of quantum mechanics instead of an infinite-dimensional one which follows from the canonical quantization and underlies the standard quantum field theory. We briefly discuss the relationship between those two approaches to field quantization. By applying the precanonical quantization to Yang-Mills fields we derive a Clifford-algebra-valued operator whose spectrum is related to the mass spectrum of quantum excitations of quantum YM field. We present a naive estimation of the distance between the lowest and the first excited state of this operator in the case of quantum $SU(2)$ YM field and call for a further development of the spectral theory of operators on hypercomplex Hilbert spaces which would allow us to treat the problems emerging from precanonical quantization of fields more rigorously.

Mostafa Mbekhta (Morocco)

Polar decomposition, Aluthge and Mean Transforms

Abstract. In this talk, we give a new proof of the existence and uniqueness of the polar decomposition of a bounded operator on a Hilbert space. We show that the polar part of $T = V|T|$ the polar decomposition of T, is given by an explicit formula :

$$V = \int_0^\infty T \exp(-sT^*T)|T|ds.$$

On the other hand, we are establishing new results on the Aluthge ($\Delta(T) = |T|^{\frac{1}{2}}V|T|^{\frac{1}{2}}$) and mean ($\widehat{T} = \frac{1}{2}[V|T| + |T|V]$) transforms of a bounded operator acting on a Hilbert space.

Posters

Souad Abumaryam (Libya)

Identity of Chebyshev Polynomial

Abstract. In this paper, we give some important fact about Chebyshev polynomial and their properties. We investigate some properties of Chebyshev polynomials arising from their orthogonality, generating function. We derive an interesting identities for these polynomials such as generating functions, relations between the two kinds of Chebyshev polynomials and integral representation. Finally, we try to involve an Umbral calculus of Chebyshev polynomial.

References

1 Mason, J. And Handscomb, D. Chebyshev Polynomials. , CRC Press, 2003.

2 Trefethen, N.Approximation Theory and Approximation Practice. , University of Oxford, 2012

3 Roman, S.The Umbral Calculus. , Academic Press, INC. Orlando, Florida, 1984.

4 Rota, G. Kung, J. And Yun, C.Combinatorics The Rota Way. , Cambridge University, 2009.

5 Araci,S. Acikgoz,M. Bagdasaryan,A. And Sen, E. The Legendre polynomials associated with Bernoulli, Euler, Hermite and Bernstein polynomials. , Turkish Journal for Analysis and Number Theory, 2013. 1

Abdelmoumene Mohamed Adib (Algeria)

Light Cone Quantization of Open and Closed String Theories in a canstant NS-B field

Abstract. The main results of a bosonic string theory in presence of a N-S B-Field are the fact that the equations of motion do not depend on the B-field while the Noether currents can depend on B as the same as the boundary conditions. In particular, the momentum P depend on the B-field. A great number of works have investigated the covariant quantization of this theory and shows that the coordinates are non-commutative and that the physical states are subject to the Virasoro conditions, which depend on the B-Field. The B-field dependence of the P momentum makes difficult the definition of the light cone gauge, in the work of Gianluca Grignani, Marta Orselli and Gordon W. Semenoff. This difficulty is resolved for the closed bosonic string when it wraps a compactified dimension on which we have the only non-zero component of the B-field. The periodicity conditions allow then the use of the light cone gauge. In this work, we study the coherence of this model through the study of the spectrum and its degeneracy compared to the development of the partition function. New physical states are possible depending on the P quantum number and the number of wrapping of the closed string on the compactified direction. In the second step, we have investigated the open string case. The absence of the periodicity condition added to the boundary conditions, which depend on the B-field, makes the light cone gauge difficult to define. We have proposed one approach based on the results of the covariant quantization and the use of the light cone coordinates system (in progress)

Mohamed Amazioug (Morocco)

Hierarchy of quantum correlations in atom-optomechanical system

Abstract. In this paper we analyze non-classical correlations between atoms and movable mirror in atom-optomechanical systems. This system consisting of an atomic ensemble placed inside an optical nanoresonator with a vibrating mirror. First, we give the Hamiltonian and the explicit expression of covariance matrix leading to the quantum equations describing the dynamic evolution of the system. Then, the non-classical correlations are quantified using the logarithmic negativity, and beyond entanglement we use Gaussian quantum discord. We propose also a scheme for examining the evolution of Gaussian quantum steering and its asymmetry. We show that the entanglement of the two mechanical modes is very strongly related to the parameters characterizing the environment where the movable mirror evolve, in particular the thermal bath temperature.

Tarik Amtout (Morocco)

Preliminary group classification of the boundary layer equations

Abstract. The preliminary group classification is used for boundary layer equations of a thermo-dependent fluid around any profile. The equations involve three parameters which are the stress, the conductivity and the pressure gradient. In this work by using optimal system of one-, two-dimensional of sub-algebras, we demonstrate the existence of constitutive laws that give rise to self-similar solutions compatible with the boundary conditions.

Abdessalem Benammar (Algeria)

Missing data restoration of sinogram in limited-angle computed tomography

Abstract. We present in this work the limited-angle computed tomography, which is an ill-posed inversion problem. This case often exists in the industry to allow faster non-destructive testing during production phase. However, the inspection is difficult to achieve due to the shape and size of the inspected parts. During the last decade, various approaches were proposed for case of limited-angle. These methods were developed for medical application use and do not take into account physical limitations specific to industrial materials. The aim of this work is to propose a method, which permits to recover the missing data in the acquisition of projections using minimizing a function. We tested our method with sinogram obtained from Shepp-Logan phantom containing missing projections. The reconstruction image of inpainted sinogram achived using FBP method and Iterative cimmino method. The results clearly show that the proposed method can retrieve accurate information that leads to a better-reconstructed image.

References

1 Yixing Huang, Guenter Lauritsch, Mario Amrehn, Oliver Taubmann, Viktor Haase, Daniel Stromer, Xiaolin Huang, Andreas Maier, "Image Quality Analysis of Limited Angle Tomography Using the Shift-Variant Data Loss Model", Bildverarbeitung für die Medizin, pp 277-282, 2016.

2 Hongliang Qi, Zijia Chen, Shuyu Wu, Yuan Xu, Linghong Zhou, "Iterative image reconstruction using modified non-local means filtering for limited-angle computed tomography", Physica Medica, Volume 32, Issue 9, pp1041-1051 September 2016.

Amin Benaissa Cherif (Algeria)

Oscillation theorems for higher order neutral nonlinear dynamic equations on time scales

Abstract. we will establish some oscillation criteria for the even-order nonlinear dynamic equation

$$\left(a\left(x^{\Delta^{n-2}}\right)^{\gamma}\right)^{\Delta^2}(t)+f\left(t,x^{\alpha}(t)\right)=0,\quad t\in[t_0,+\infty)_{\mathbb{T}} \tag{7}$$

on a time scales $\mathbb{T}$ with n is an even integer ≥ 3, where γ and α are the ratios of positive odd integer and a is areal valued rd-continuous function defined on $\mathbb{T}$.

Iness Haouala (Tunisia)

Polynômes de Jacobi-Dunkl et leur analyse harmonique

RÉSUMÉ

On s'intéresse à l'analyse harmonique associée à l'opérateur de Jacobi-Dunkl sur $]-\pi/2,\pi/2[$. On rappelle, tout d'abord, quelques résultats sur les polynômes de Jacobi et on donne une représentation intégrale de Laplace pour les polynômes de Jacobi-Dunkl. Ensuite, on présente la transformation de Jacobi-Dunkl ainsi que ses propriétés. Finalement, on introduit l'opérateur de translation généralisée qui nous servira à définir le nouveau produit de convolution dans ce cadre.

Abstract. We are concerned with the harmonic analysis associated with the Jacobi-Dunkl operator on $]-\pi/2,\pi/2[$. We recall, first of all, some results on Jacobi polynomials and we give an integral Laplace representation of the Jacobi-Dunkl polynomials. Then, we present the Jacobi-Dunkl transform and some of its properties. Finally, we introduce the generalized translation operator, which will be used to define a new convolution product.

Tomas Kalvoda (Czech Republic)

New Family of Symmetric Orthogonal Polynomials and a Solvable Model of a Kinetic Spin Chain

Tomas Kalvoda

Abstract. We study an infinite one-dimensional Ising spin chain where each particle interacts only with its nearest neighbors and is in contact with a heat bath with temperature decaying hyperbolically along the chain. The time evolution of the magnetization (spin expectation value) is governed by an infinite Jacobi matrix whose spectral problem leads us to a new two-parameter family of orthogonal polynomials expressible in terms of basic hypergeometric (or q-hypergeometric) series. We then study the properties of this family of orthogonal polynomials. Finally, we return to the Ising model and study the magnetization time evolution. This is joint work with Frantisek Stampach. chain. The time evolution of the magnetization (spin expectation value) is governed by an infinite Jacobi matrix whose spectral problem leads us to a new two-parameter family of orthogonal polynomials expressible in terms of basic hypergeometric (or q-hypergeometric) series. We then study the properties of this family of orthogonal polynomials. Finally, we return to the Ising model and study the magnetization time evolution. This is joint work with Frantisek Stampach.

Fatima Zohra Ladrani (Algeria)

Oscillation theorems for fourth-order nonlinear Dynamic equations on time scales

Abstract. we will establish some oscillation criteria for the fourth-order nonlinear dynamic equation on time scales

$$\left(p\left(x^{\Delta}\right)^{\alpha}\right)^{\Delta^{3}}(t)+q(t)\left(p\left(x^{\Delta}\right)^{\alpha}\right)^{\Delta^{2}}(t)+f\left(t,x(\tau(t))\right)=0 \tag{8}$$

on a time scales, where is α quotient of odd positive integer and >0.

Schedule of talks

Monday, April 1

Time	Activity	Chair
7:30 - 9:00	Registration	
9:00 - 10:00	Opening	
10:00 - 10:30	Coffee	
10:30 - 11:10	**G. Hans Feichtinger**, Coherent states, Gabor multipliers, and the Banach Gelfand Triple	J. P. Gazeau
11:10 - 11:50	**Pavel Exner**, On Schrödinger operators exhibiting a parameter-dependent spectral transition	
11:50 - 12:30	**Omar El-Fallah**, Estimates of the singular values of Hankel operators on Bergman spaces	
12:30 - 14:00	Lunch	
14:00 - 14:40	**Hélène Airault**, Matrix operators on symmetric Kaehler domains	A. Vourdas
14:40 - 15:20	**Nelson Faustino**, Fractional Fokker-Planck equations on lattices associated to Wilson-Dirac type operators	
15:20 - 16:00	**Abdenacer Makhlouf**, q-Deformations and twisted algebraic structures	
16:00 - 16:30	Coffee Break	

Parallel Sessions

Time	Activity / Room A	Chair
16:30 - 17:00	**Jasel Berra-Montiel**, Towards a representation of Loop Quantum Gravity in the Deformation Quantization approach	A.I. Zayed
17:00 - 17:30	**Alberto Molgado**, Polysymplectic structures and general relativistic models	
17:30 - 18:00	**Milton dos Santos Ferreira**, Gyroharmonic analysis and the continuous wavelet transform on the ball	
	Activity / Room B	
16:30 - 17:00	**Fouzia El Wassouli**, Wigner transform on line bundle on bounded symmetric spaces	E. Curado
17:00 - 17:30	**Nadia Ourchane**, Characterization of the L^p-range of the Poisson transform on the Octonionic Hyperbolic plane	
17:30 - 18:00	**Francesco Gargano**, Bi-coherent and Bi-squeezed states in non Hermitian Quantum Mechanics	

Tuesday, April 2

Time	Activity	Chair
8:30 - 9:10	**Jacques Faraut**, Horn's problem from a probabilistic point of view	**H. Feichtinger**
9:10 - 9:50	**Uwe Kähler**, Clifford-Krein modules with reproducing kernels	
9:50 - 10:30	**Danail Brezov**, On the Complex Kinematics of Relativity	
10:30 - 11:00	Coffee Break	
11:00 - 11:40	**Ali Baklouti**, Harmonic Analysis around monomial representations of exponential solvable Lie groups	**P. Balazs**
11:40 - 12:20	**Apostol Vourdas**, Fermionic quantum computation and exterior calculus	
12:20 - 13:00	**Azzouz Awane**, Polarized k-symplectic geometry	
13:00 - 14:30	Lunch	
14:30 - 15:10	**Ahmed I. Zayed**, Sampling on a disc for a class of linear canonical transforms	**S. Berceanu**
15:10 - 15:50	**Fabio Bagarello**, Extended kq-representation and bi-coherent states	
15:50 - 16:30	Coffee Break	

Parallel Sessions

Time	Activity / Room A	Chair
16:30 - 17:00	**Achraf Ouald Chaib**, The Poisson transform on homogeneous vector bundles over $Sp(n,1)/Sp(n) \times Sp(1)$	**N. Faustino**
17:00 - 17:30	**Vladimir Jaćimović**, Quasiconformal geometry of a magnetic field	
17:30 - 18:00	**Mazouz Amel**, Husimi distribution: correspondence between quantum and classical dynamics	
	Activity / Room B	
16:30 - 17:00	**Kamal Diki**, The slice hyperholomorphic Cholewinski-Fock space on quaternions	**F. Bagarello**
17:00 - 17:30	**Youssef Omari**, Riesz bases of reproducing kernels in small Fock spaces	
17:30 - 18:00	**Noureddine Imesmad**, A characterization of the L^2-range of the Poisson transform on homogeneous line bundles over the matrix ball	
18:00 - 20:00	Honoring the memory of Professor Ahmed Intissar	

Wednesday, April 3

Time	Activity	Chair
8:30 - 9:10	**Mostafa Mbekhta**, Polar decomposition, Aluthge and Mean Transforms	**T. Shirai**
9:10 - 9:50	**Jerzy Lukierski**, New Approach to colour symmetries: Dirac equations for quarks and algebraic confinement	
9:50 - 10:30	**Nobukazu Shimeno**, Matrix valued spherical functions on real semisimple Lie groups	
10:30 - 11:00	Coffee Break	
11:00 - 11:40	**Khalid Koufany**, Conformally covariant bi-differential operators for differential forms	**A. Baklouti**
11:40 - 12:20	**Sébastien Tremblay**, On generalized powers in one-dimensional supersymmetric quantum mechanics	
12:20 - 13:00	**Evaldo M. F. Curado**, Optimizing Helstrom bound with non-standard coherent states	
13:00 - 14:30	Lunch	
14:30	Excursion	

Thursday, April 4

Time	Activity	Chair
8:30 - 9:10	**Anatol Odzijewicz**, A family of integrable perturbed Kepler systems	
9:10 - 9:50	**Georgi Raikov**, Resonances and SSF singularities for a magnetic Schrödinger operator	**P. Exner**
9:50 - 10:30	**Vladimir V. Kisil**, Hidden aspects of coherent states from group representation	
10:30 - 11:00	Coffee Break	
11:00 - 11:40	**Hashim A. Yamani**, Supersymmetry of tridiagonal Hamiltonians and their associated orthogonal polynomials	
11:40 - 12:20	**Mariano A del Olmo**, Special functions, Lie algebras and rigged Hilbert spaces	**U. Kähler**
12:20 - 13:00	**Peter Balazs**, Frames and Operators	
13:00 - 14:30	Lunch	
14:30 - 15:10	**Stefan Berceanu**, Differential geometry on homogeneous spaces attached to the Jacobi group	**L. J. Päivärinta**
15:10 - 15:50	**Paula Cerejeiras**, $SU(3)$ symmetries on generalised ternary Clifford algebras	
15:50 - 16:30	Coffee Break	
19:30	Conference dinner at *Chez Ali*	

Friday, April 5

Time	Activity	Chair
8:30 - 9:10	**Jean P. Gazeau**, Various generalizations of Glauber-Sudarshan coherent states for Quantum Optics	
9:10 - 9:50	**Tomoyuki Shirai**, Limit theorems for determinantal point processes	**A. Odzijewicz**
9:50 - 10:20	Coffee Break	
10:20 - 11:00	**Lassi J.Päivärinta**, Scattering of acoustic and electromagnetic waves by screens	**M. del Olmo**
11:00 - 11:40	**Luís D. Abreu**, Planar electrostatics in higher Landau levels	

Parallel Sessions

Time	Activity / Room A	Chair
12:00 - 12:30	**Dmitry Shirokov**, Method of averaging in Clifford algebras and applications	**P. Cerejeiras**
12:30 - 13:00	**Briceyda B. Delgado**, A right inverse operator for curl$+\lambda I$ and applications	
13:00 - 15:00	Lunch	
15:00 - 15:30	**Edwin Beggs**, Geodesics in noncommutative geometry	
15:30 - 16:00	**Abdelhadi Benahmadi**, Non-trivial 1d and 2d Segal-Bargmann transforms	**V. V. Kisil**
16:00 - 16:30	Coffee Break	
16:30 - 17:00	**Pedro Alberto**, Mathematical solutions of Coulomb type tensor interactions in relativistic quantum mechanics	
17:00 - 17:30	**Abdessamad Belfakir**, Generalized Heisenberg algebra: periodicity and finite representation	**G. Raikov**
17:30 - 18:00	**Igor Kanatchikov**, On the spectrum of a hypercomplex quantum operator related to the Yang-Mills mass gap	
	Activity / Room B	
12:00 - 12:30	**Zakariyae Mouhcine**, An Integral Transform Connecting Spherical Analysis on Harmonic NA Groups to that of Odd Dimensional Real Hyperbolic Spaces	
12:30 - 13:00	**Yahyeh Souleiman**, Numerical Analysis of a Sliding frictional contact problem with Unilateral Contact and Normal Compliance	**S. Tremblay**
13:00 - 15:00	Lunch	
15:00 - 15:30	**Mohammed Berkani**, On the B-discrete spectrum	**D. Brezov**
15:30 - 16:00	**Aneta Sliżewska**, Banach-Lie groupoids related to W^*-algebra	
16:00 - 16:30	Coffee Break	
16:30 - 17:00	**Abdoul Salam Diallo**, Affine Szabó manifolds	
17:00 - 17:30	**Zaiem Slimane**, Schrödinger Differential equation and Non-commutative geometry	**L. D. Abreu**
17:30 - 18:00	**Mohammed Ziyat**, Landau level coherent state transforms for quasi-tori	

Acknowledgments

The organisers of the ICMMP - International Conference on Mathematical Methods in Physics wish to thank the many Moroccan companies who's financial support was crucial for the success of this event:

- SCR - Société Centrale de Réassurance
- CARC - The African Centre for Catastrophe Risks
- CAT - Company d'Assurance Transport
- AXELI - Services Informatique et infogérance
- ANRT - Agence Nationale de Réglementation des Télécommunications
- GASUP - Le Groupement d'Assurances du Supérieur
- SARL - Work Bureau-Béni Mellal

Our sincere gratitude goes also to the Research Center CIDMA, University of Aveiro in Portugal, to Sultan Moulay Slimane University, and to the Ibn Tofail University for their financial support. Furthermore, our deepest thanks goes to the Faculty of Sciences and Technics of Béni Mellal which has provided us with both financial and logistical support.

Last but certainly not least, the conference would not have been possible without the great support of the Riyad Mogador Kasbah Hotel which, by their logistical and administrative support, and the continuous help of its staff members, greatly facilitated the travel and stay of the participants.


ANRT
الوكالة الوطنية لتقنين المواصلات
ⵜⴰⵙⴷⴰⵡⵜ ⵜⴰⵏⴰⵎⵓⵔⵜ ⵏ ⵜⵙⵎⵓⵔⵜ ⵏ ⵜⵉⵍⵉⵎⵓⵙⵉⵏⵜ
Agence Nationale de Réglementation des Télécommunications