

Lemma (1) Sign does not depend on the choice of local ori. of  $N^3$  at  $y$ .

(2) If we change the ori. of  $M^4$ , then the sign changes.

Theorem (T. Yamamoto - S)

$f : M^4 \rightarrow N^3 \in C^\infty$  stable  
 ↑ closed oriented

$\Rightarrow \| \text{III}^d(f) \| = \text{signature of } M^4 \quad (*)$

Idea for the Proof

- Both  $\| \text{III}^d(f) \|$  & sign. of  $M^4$  are ori. cob. inv.
  - ori. cob. grp  $\Omega_4 \xrightarrow[\cong]{\text{sign.}} \mathbb{Z}$
- $\Rightarrow$  suffices to check  $(*)$  for a generator. //