<u>Theorem</u> $f: M^2 \to \mathbb{R}$ stable Morse fct

$\Rightarrow \chi(M^2) \equiv |\tilde{I}^2(f)| \pmod 2$

In fact, $\hat{\alpha}_2' = [\tilde{I}_0^2 + \tilde{I}_e^2]$ is a

generator of $H^1(\underbrace{\tau(3,2)}_{\text{fibers of stable maps}}, \mathcal{P}_{3,2}(2))$

$\Rightarrow$ $\mathfrak{N}_2$: 2-dim. unoriented cobordism grp.

$$\begin{array}{ccccc} \Phi : & \mathfrak{N}_2 & \longrightarrow & \mathbb{Z}_2 & \\ & \Psi & & \Psi & \\ & [M] & \longmapsto & |\tilde{I}^2(f)| & \text{for } f: M \to \mathbb{R} \end{array}$$

well-defined homomorphism

On the other hand

$$\begin{array}{cccc} \Phi' : & \mathfrak{N}_2 & \xrightarrow{\cong} & \mathbb{Z}_2 \\ & \Psi & & \Psi \\ & [M] & \longmapsto & \chi(M) \bmod 2 \end{array}$$

We can check $\Phi(\mathbb{R}P^2) = \Phi'(\mathbb{R}P^2) = 1$.

$\Rightarrow \Phi = \Phi'$