§7. Application to Map Germs

Def. $g, g' : (\mathbb{R}^3, 0) \to (\mathbb{R}^2, 0)$ C^∞ map germs

topologically $\mathcal{A}$-equiv.

$\overset{\text{DEF}}{\Longleftrightarrow}$

$$\begin{array}{ccc} (\mathbb{R}^3, 0) & \xrightarrow{\exists\, \Phi} & (\mathbb{R}^3, 0) \\ g \downarrow & \circlearrowleft & \downarrow g' \\ (\mathbb{R}^2, 0) & \xrightarrow{\exists\, \varphi} & (\mathbb{R}^2, 0) \end{array}$$

Φ, φ : homeo.

topologically $\mathcal{A}_+$-equiv. if φ preserves the ori. of $\mathbb{R}^2$

Def. $g : (\mathbb{R}^3, 0) \to (\mathbb{R}^2, 0)$ generic

$\overset{\text{DEF}}{\Longleftrightarrow}$ for $0 < \varepsilon \ll \delta \ll 1$

$$\begin{cases} D^3_\delta \cap g^{-1}(S^1_\varepsilon) : C^\infty \text{ mfd with boundary} \\ g_\partial = g|_{D^3_\delta \cap g^{-1}(S^1_\varepsilon)} : D^3_\delta \cap g^{-1}(S^1_\varepsilon) \to S^1_\varepsilon : C^\infty \text{ stable} \\ g|_{\partial D^3_\delta \cap g^{-1}(D^2_\varepsilon)} : \partial D^3_\delta \cap g^{-1}(D^2_\varepsilon) \to D^2_\varepsilon : \text{submersion} \end{cases}$$