

(1) suspension of any element
of $\mathcal{T}(n, p)$ belongs to $\mathcal{T}(n+1, p+1)$

(2) equiv. w.r.t. $P_{n, p}$

\Rightarrow suspensions are equiv. w.r.t. $P_{n+1, p+1}$

\Downarrow

$$\exists S_K^\# : C^K(\mathcal{T}(n+1, p+1), P_{n+1, p+1}) \rightarrow C^K(\mathcal{T}(n, p), P_{n, p})$$

Cochain map

i.e.
$$\delta_K \circ S_K^\# = S_{K+1}^\# \circ \delta_K$$