

Exercises

1. Let F_n be a closed, orientable surface with genus $n (\geq 1)$.

Find $H_*(F_n)$.

2. Show that if d is an integer, and $n \geq 1$, then there is a map $R: S^n \rightarrow S^n$ of degree d .

3. Let $L(n, k)$ be the quotient space of the ball B^3 as follows: Any point $(z, t) \in B^3$, where z is a complex number, t is a real, and $|z|^2 + |t|^2 \leq 1$.

Let $\lambda = \exp(2\pi i/n)$. Define $f: S^2 \rightarrow S^2$

by $f(x) = (\lambda^k z, -t)$.

Identify each point $x = (z, t)$ of the lower hemisphere E_-^2 of $\partial B^3 = S^2$ with $f(x)$ of the upper hemisphere E_+^2 . The resulting quotient space is called the lens space $L(n, k)$.

Find $H_*(L(n, k))$.